Math 23 **Exam 1** **Name:**

29 September 1997 100 Points

"Show enough work to justify your answers."

1. Two systems of equations are given, along with the echelon forms of their augmented matrices. For each system, write the augmented matrix for the original system, the equations that correspond to the reduced system, and the solutions. (10 points each)

(a)
$$\begin{array}{rcrcrcrcr} 3x & - & y & & & + & 2w & = & 1 \\ 2x & + & y & + & 2z & - & w & = & 0 \\ 5x & & & + & 2z & + & w & = & 1 \\ x & - & 2y & - & 2z & + & 3w & = & 1 \end{array} \qquad \begin{pmatrix} 1 & 0 & 2/5 & 1/5 & 1/5 \\ 0 & 1 & 6/5 & -7/5 & -2/5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(b)
$$\begin{array}{rcrcrcr} -3x & + & y & - & 3z & = & 2 \\ 2x & + & 4y & - & 2z & = & 6 \\ -x & + & 5y & - & 5z & = & -1 \end{array} \qquad \begin{pmatrix} 1 & 0 & 5/7 & 0 \\ 0 & 1 & -6/7 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. Show that the following matrices are inverses. (5 points)

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 2 & -1 \\ 2 & 3 & 0 \end{pmatrix} \qquad \frac{1}{7}\begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 3 \\ 2 & -5 & 4 \end{pmatrix}$$

3. Solve the following systems. Look at them all carefully before you start. (15 points)

$$\begin{array}{rcrcrcr} x & - & y & + & z & = & 1 \\ 2x & + & 2y & - & z & = & -3 \\ 2x & + & 3y & & & = & 2 \end{array} \qquad \begin{array}{rcrcrcr} x & - & y & + & z & = & 0 \\ 2x & + & 2y & - & z & = & 2 \\ 2x & + & 3y & & & = & -1 \end{array} \qquad \begin{array}{rcrcrcr} x & - & y & + & z & = & 3 \\ 2x & + & 2y & - & z & = & 2 \\ 2x & + & 3y & & & = & 1 \end{array}$$

4. Do any **three** of the following proofs. If you work on more than three, I will use the best three to compute your score. (10 points each)

   (a) Suppose A and B are square matrices of the same dimension. Prove that if $(A+B)^2 = A^2 + 2AB + B^2$, then A and B commute.

   (b) Suppose that A is a square matrix and prove that AA^T is symmetric.

   (c) Suppose that A is a 10×10 matrix and c is a scalar. Prove that $\det(cA) = c^{10} \det A$.

   (d) Suppose that A and B are invertible matrices of the same dimension. What is the inverse of AB? Prove your assertion.

   (e) A problem on one of your problem sets was to prove that if A and B are square matrices of the same size and B is singular, then $C = AB$ is singular. Here is a hint that is easier than the one suggested in the book: prove it by contradiction, namely, suppose that C is nonsingular, that is, C^{-1} exists. Use this to find an inverse for B.

5. Evaluate the following determinant. (15 points)

$$\det \begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & -1 & 1 & 4 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 4 \end{pmatrix}$$

6. Let $V = \{(x, y) \in \mathbf{R}^2 \mid xy = 0\}$. Show that V is closed under one of the vector space operations (vector addition and scalar multiplication) but not the other. Is V a subspace of $\mathbf{R}^2$? (15 points)