Linear Functions, Independence, and Spanning

Math 223, Spring 2010

Suppose that V and W are vector spaces and that $T : V \to W$ is linear. Let $\mathbf{v}_1, \ldots, \mathbf{v}_n \in V$, and let $\mathbf{w}_1 = T(\mathbf{v}_1)$, …, $\mathbf{w}_n = T(\mathbf{v}_n)$.

The following facts are similar, but not the same. You should be able to prove them.

- If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are independent and T is invertible, then $\mathbf{w}_1, \ldots, \mathbf{w}_n$ are independent.
- If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are independent and T is one-to-one, then $\mathbf{w}_1, \ldots, \mathbf{w}_n$ are independent.
- If $\mathbf{w}_1, \ldots, \mathbf{w}_n$ are independent, then $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are independent.
- If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ span V, then $\mathbf{w}_1, \ldots, \mathbf{w}_n$ span $\mathrm{im}(T)$.
- If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ span V and T is onto, then $\mathbf{w}_1, \ldots, \mathbf{w}_n$ span W.

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