THE PRINCIPLE OF MATHEMATICAL INDUCTION

If you are like me, mathematical induction makes you feel a little uneasy. It's a bit too magical, like getting something for nothing. Most treatments seem to want you to take it on faith, but its proof is actually quite easy. Here it is. ($\mathbb{N}$ denotes the set of all positive integers.)

Theorem (Principle of Mathematical Induction). *Let S_n be a statement about $n \in \mathbb{N}$. If*

(1) *S_1 is true, and*
(2) *For each $n \in \mathbb{N}$, the truth of S_n implies the truth of S_{n+1},*

then S_n is true for all $n \in \mathbb{N}$.

Proof. The proof is by contradiction: suppose that it's not the case that S_n is true for all $n \in \mathbb{N}$. Then S_n must be false for at least one $n \in \mathbb{N}$. There may be several n's for which S_n is false. Let N be the smallest one.

There are two possibilities: either $N = 1$ or $N > 1$. But if $N = 1$, then we are saying that S_1 is false, which contradicts the first assumption above. Thus N is not 1; it must be greater than 1.

Consider the statement S_{N-1}, which is possible since $N > 1$. Now, since N is the *smallest* value of n for which S_n is false, then S_{N-1} must be true. But then the truth of S_{N-1} doesn't imply the truth of S_N, which contradicts the second assumption above.

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