Math 14 **Exam 3** **Foote**

14 April 1999 100 Points

"Show enough work to justify your answers."

READ THIS !!! You are to work the problems 1 and 2, and then any four of problems 3 through 8.

When you use *Mathematica* as an essential part of solving a problem, indicate in some detail how you use it, enough so it is clear to me how you draw your conclusions. If in doubt about how much to say, please ask. You do not need to indicate the use of *Mathematica* if you use it simply to check your work. If you have trouble with *Mathematica*, please ask.

Remember that some integrals are more easily evaluated by changing coordinates or changing the order of integration.

1. Evaluate $\int_0^2 \int_0^{y^2} \frac{1}{1+y^3}\, dx\, dy$ showing all steps. (10 points)
2. The graph of $y^2 = x^2 + z^2 - 1$ is shown. In each part, set the appropriate variable to a constant, determine the resulting equation, sketch it on the surface, and say if it is a parabola, circle, ellipse, or hyperbola. Clearly label the curves you add to the picture. (10 points)

   a) $y = -2$ b) $z = 2$

   READ THIS !!! Do any **four** of the remaining problems. If you work on more than four, you will get credit for the best four. (20 points each)
3. For each function, determine which picture is a plot of some of its level curves, and briefly explain how you know it.

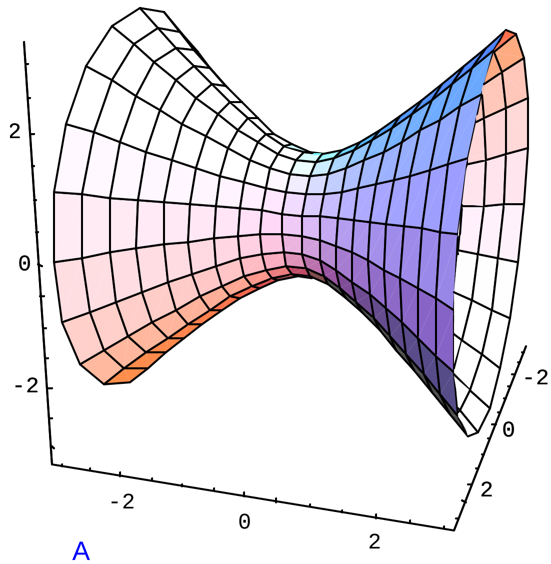
   a) $f(x,y) = x^2 + y^2$ c) $g(x,y) = x^2 - y^2$

   b) $h(x,y) = x^2 + 2y$ d) $k(x,y) = x + 2y$
4. Consider the surface $x^2 + xy + y^2 - z^2 = 9$ and the points $(3, 1, -2)$ and $(-3, -1, 2)$, which are on the surface.

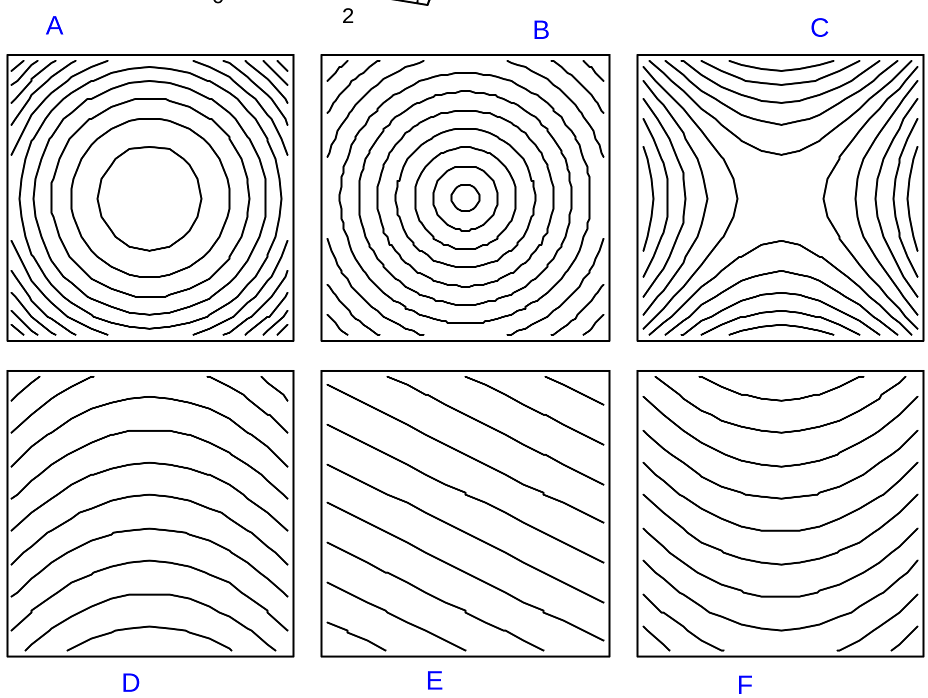
   a) Find equations of the planes that are tangent to the surface at these two points. (15 points)

   b) Explain why these tangent planes are parallel. Hint: Multiply the equations out (don't multiply them together!) and use the results to argue that no point can satisfy both equations. (5 points)
5. Let R be the region that is the upper half of the inside the cardioid $r = 1 + \cos\theta$ and evaluate the integral $\iint_R \sqrt{x^2+y^2}\, dA$ by converting it to polar coordinates. (Recall that $x = r\cos\theta$ and $y = r\sin\theta$.)
6. Find all critical points of $f(x,y) = y^2 - 2xy + \frac{x^3}{3} - 3x$ and determine if they are local minima, local maxima, or saddle points.
7. Sketch the region of integration and switch the order of integration for the following double integral. Do not evaluate.

   $$\int_0^2 \int_0^{4-2x} x^2 e^{xy}\, dy\, dx$$
8. A rectangular box has a volume of 10 cubic feet. The material for the top costs \$3 per square foot. The material for the other five sides costs \$2 per square foot. Find the dimensions *and the cost* of the least expensive box.



Problem 2



Problem 3

Answers and hints to selected problems.

1. See what *Mathematica* gets.
2. One of them is a circle, the other a hyperbola.
3. Set each function to a constant and think about the graph of the resulting equation in x and y. This will suggest one or more of the choices and eliminate others.
4. a) $7(x-3)+5(y-1)+4(z+2)=0$ and $7(x+3)+5(y+1)+4(z-2)=0$
5. You need to sketch the curve. $5\pi/6$
6. $(-1,-1)$ is a saddle point; $(3,3)$ is a local minimum.
7. The only way to do this is to sketch the region. The function being integrated has nothing to do with switching the order of integration.
8. The length and width are both 2 feet. The height is $5/2=2.5$ feet.