Math 14 **Exam 1** **Name:** ______________________________

8 February 1999 100 Points

"Show enough work to justify your answers."

1. Evaluate the following antiderivatives. Show all steps—don't just give answers. (10 points each)

 a) $\displaystyle\int \frac{e^x}{(e^x+1)^2}\,dx$

 b) $\displaystyle\int x\cos(2x)\,dx$

 c) $\displaystyle\int \frac{1}{x+\sqrt{x}}\,dx$ Let $x = u^2$.

2. Evaluate the antiderivative $\displaystyle\int \frac{1}{4-x^2}\,dx$ in two ways (10 points each):
 a) Factor the denominator and use partial fractions.
 b) Use the substitution $x = 2\sin\theta$ and use integral formula #17 in the table in the book.

$$\left.\begin{array}{l}|I-L_n|\\ |I-R_n|\end{array}\right\} \le \frac{K_1(b-a)^2}{2n}, \quad |I-T_n| \le \frac{K_2(b-a)^3}{12n^2}, \quad |I-M_n| \le \frac{K_2(b-a)^3}{24n^2}, \quad |I-S_n| \le \frac{K_4(b-a)^5}{180n^4}$$

In the following problem, when you use *Mathematica*, indicate how you use it.

3. Consider the integral $\displaystyle\int_0^3 \frac{1}{x+e^x}\,dx$.
 a) Give a midpoint sum approximation for this integral using 100 intervals. (5 points)
 b) Use an error estimate formula to find an upper bound on the error of the approximation you gave in part a). (12 points)
 c) Use an error estimate formula to determine how many intervals are needed to guarantee a Simpson's Rule approximation of this integral with an error not to exceed $10^{-4} = .0001$. (13 points)

4. Consider $\int_0^1 \ln x\,dx$ and $\int_0^\infty e^{-2x}\,dx$.
 a) Explain why each integral is improper and re-write as a limit of a proper integral. (10 points)
 b) Evaluate the second integral. (10 points)

5. (Extra Credit) Draw a picture illustrating a situation in which a left sum is more accurate than a trapezoid sum. (5 points)

Answers and hints to selected questions.

1. See what *Mathematica* gets.
2. Antiderivative two ways.
   a) The *Mathematica* command **Apart** will do partial fraction decompositions. The antiderivative you get is $\frac{1}{4}\ln|2+x| - \frac{1}{4}\ln|2-x| + C$.
   b) The antiderivative you get this time is $\frac{1}{2}\ln\left|\frac{2}{\sqrt{4-x^2}} + \frac{x}{\sqrt{4-x^2}}\right| + C$. It's a good algebra exercise to show that the answers in parts a) and b) are equal! You could also see if *Mathematica* will show they are equal. Try subtracting them and then use //**Simplify** if necessary.
3. Numerical approximation of integral.
   a) $\int_0^3 \frac{1}{x+e^x}\,dx \approx M_{100} = .760459$
   b) You can take $K_2 = 7$, in which case the error bound is .0008.
   c) You can take $K_4 = 261$, in which case the error is guaranteed not to exceed .0001 when $n \geq 44$.
4. Improper integrals.
   a) The first integral is improper because $\ln x$ has no limit as $x \to 0^+$. It is rewritten as $\lim_{a\to 0^+} \int_a^1 \ln x\,dx$.