Math 14 **Final Exam, Version A** **Foote**

5 May 1998 200 Points

"Show enough work to justify your answers."

READ CAREFULLY! Do all parts of problems 1 (10 points each). Then do any **ten** of the remaining problems (15 points each).

If you use *Mathematica* in a problem, be sure to say so, and say briefly what you did. You should include relevant output on your exam. If you have problems with *Mathematica*, please let me know. I can't give credit for nonsense answers it produces. If you are sharing a computer, when you finish using *Mathematica* for a problem, please close the window you used (you don't have to shut down *Mathematica*).

Possibly useful formulas: $|I - T_n| \leq \dfrac{K_2(b-a)^3}{12n^2}$ $|I - M_n| \leq \dfrac{K_2(b-a)^3}{24n^2}$

1. Warm up problems. Do all parts. Do by hand. (10 points each)

   a) Evaluate $\displaystyle\int_0^{\sqrt{15}} x\sqrt{x^2+1}\,dx$.

   b) Evaluate $\displaystyle\int_{-1}^{1} xe^{2x}\,dx$.

   c) What is the sum of the following series? Explain.
   $1 + \ln 2 + \dfrac{(\ln 2)^2}{2!} + \dfrac{(\ln 2)^3}{3!} + \dfrac{(\ln 2)^4}{4!} + \dots$

   d) Let $f(x,y) = \sin(xy^2)$ and compute df.

   e) Evaluate $\displaystyle\int_0^{1/2}\int_0^{\pi x} \sin y\,dy\,dx$.

READ THIS!! Do any **ten** of the following problems (15 points each). If you work on more than ten you will get credit for the best ten.

2. Explain why the following integral is improper. Evaluate it by hand, showing all steps.
$$\int_0^1 \ln x\,dx$$

3. Use a trapezoid sum to approximate the following integral with an error less than 0.0001. Be clear how you determine the number of intervals to use. You may **not** use *Mathematica*'s approximation of the integral to do this. Instead, use the error estimate formula. You may use *Mathematica* to evaluate the approximating trapezoid sum.
$$\int_0^2 e^{-x^2}\,dx$$

4. Approximate the integral in the previous problem with the same degree of accuracy by writing the function as a power series. Be clear how you determine the number of terms to use. You may **not** use *Mathematica*'s approximation of the integral to do this.

5. Find the volume under the graph of $z = \sqrt{x^2+y^2}$ over the disk $x^2+y^2 \leq 9$. Work all integrals by hand.

6. The *centroid*, or geometric center, of a region R in the xy-plane is the point $(\bar{x}, \bar{y})$, where $\bar{x}$ and $\bar{y}$ are the average x and y coordinates of points in the region. More precisely,
$$\bar{x} = \frac{1}{A}\iint_R x\,dA \qquad \text{and} \qquad \bar{y} = \frac{1}{A}\iint_R y\,dA,$$
where A is the area of R. Find the centroid of the region under the pictured arch of the cosine function. Plot the centroid on the picture.

7. Explain why the following series converges. Then approximate it with an error less than 0.0001. Be clear how you determine the number of terms to use. You may **not** use *Mathematica*'s approximation of the series to do this, but you may use it to add finitely many terms.
$$\sum_{k=1}^{\infty} \frac{2^k}{3^k + \ln k}$$

8. Evaluate the following antiderivative by hand, showing all steps. $\displaystyle\int \frac{1+x}{x(1+x^2)}\,dx$

9. The picture shows level curves for a function f. Use this to determine a Riemann sum approximation for $\int_0^2 \int_0^1 f(x,y)\,dy\,dx$ with six subrectangles. Say briefly how you chose the values to use for the terms of the sum.

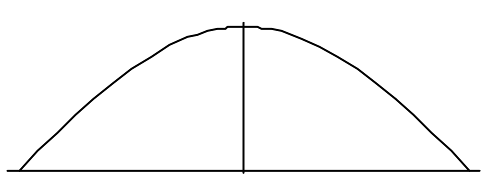
10. A manufacturer is packaging cheese wedges in the shape pictured (see MVC Section 4). If the volume of the wedge is 27 cubic inches, find the values of r, θ, and h that give the least surface area. (The area of the base is $\frac{1}{2}r^2\theta$ and the area of the curved part of the surface is $r\theta h$.)

11. Use the method of Lagrange multipliers to find the maximum of $f(x,y) = xy$ on the ellipse $x^2 + 4y^2 = 8$.

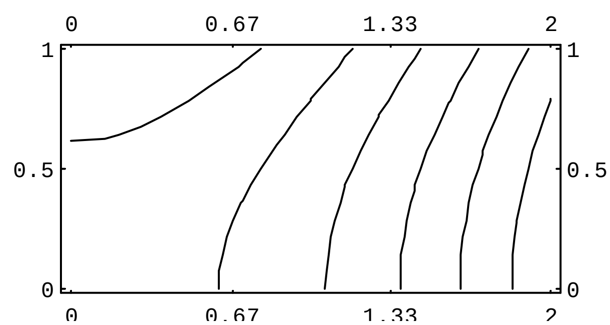
12. Consider the graph of $f(x) = x^2$ from $x = 0$ to $x = 3$. Find (approximately) the point on the curve that is halfway between the endpoints.

13. Suppose f satisfies $f'(x) = (x-1)f(x)$ and $f(0) = 1$. Find first five terms of the Maclauren series for f and use this to approximate $f(.5)$.

14. Determine if the following series converges. Note: It's not alternating and it's not a p-series!
$$1 + \frac{1}{2} - \frac{1}{9} + \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \frac{1}{49} + \frac{1}{64} - \ldots$$



Problem 6



Problem 9