Math 14 **Exam 3** **Foote**

24 April 1998 100 Points

"Show enough work to justify your answers."

You may use *Mathematica* to evaluate any integral you are not told to do by hand. Exact answers are expected unless the integral involves functions we have not studied. If you use *Mathematica* to evaluate an integral, be sure to say so, and write down the integral on your exam. If you have problems with *Mathematica*, please let me know. I can't give credit for nonsense answers it produces. If you are sharing a computer, when you finish using *Mathematica* for a problem, please close the window you used (you don't have to shut down *Mathematica*).

Remember that some integrals are more easily evaluated by changing coordinates or changing the order of integration.

1. Evaluate the following integral by hand. (10 points) $\int_0^4 \int_0^y \sqrt{y^2+9}\,dx\,dy$.

   READ CAREFULLY! Do any **SIX** of the problems 2–11. If you work on more than six, you will get credit for the best six. (15 points each)

2. We have seen that some volumes can be computed by integrating the cross-sectional area function. Explain in your own words why this works.

3. Consider the polar curve $r = \cos 3\theta$. Use a double integral to find the area of one loop of the curve.

4. The following table gives values of a function f for $0 \le x \le 1$, $0 \le y \le .5$. Use these values to give an approximation of $\int_{0.2}^{0.4} \int_{0.4}^{0.8} f(x,y)\,dx\,dy$. Show enough detail so I can tell what you are doing. In particular, draw the region of integration in the xy-plane (separate from the table), and clearly indicate the subrectangles you are using, their dimensions, and the value of the function you are using for each subrectangle.

y						
0.5	0.50	0.53	0.64	0.78	0.94	1.12
0.4	0.40	0.44	0.56	0.72	0.89	1.08
0.3	0.30	0.36	0.50	0.67	0.85	1.04
0.2	0.20	0.28	0.44	0.63	0.82	1.02
0.1	0.10	0.22	0.41	0.60	0.80	1.00
0.0	0.00	0.20	0.40	0.60	0.80	1.00
	0.0	0.2	0.4	0.6	0.8	1.0 x

5. Set up a double integral that gives the volume under $z = 4 - x^2 - y^2$ and above the xy-plane. Evaluate the integral by hand.

6. Reverse the order of integration in $\int_0^4 \int_0^{e^{2x}} x \sin y\,dy\,dx$.

7. Evaluate $\iint_R (x^2 - y)\,dA$, where R is the region bounded by the curves $y = x^2$ and $y = x^3$.

8. The base of a solid is the region under the first arch of the sine function. Every cross section of the solid perpendicular to the x-axis is a square. Find the volume of the solid.

9. The region between the curves $y = x/2$ and $y = \sqrt{x}$ is rotated around the x-axis. Find the volume generated.

10. A ten-foot chain weighing twenty pounds is coiled up on the ground. Find the work done in lifting the chain by one end until the other end is just touching the ground.

11. Write an integral that gives the length of the graph of $f(x) = x^2$ from $x = 0$ to $x = 3$. What exact value does *Mathematica* give for this integral? What approximation?

12. **Extra Credit.** (Does not count as one of the six.) Find the point on the curve in the previous problem that is halfway between the endpoints, as measured along the curve. (10 points)

Answers and hints to selected problems.

1. See what *Mathematica* gets.

3. You need to sketch the curve. $\pi/12$

4. There is no single correct answer, which is why it's important to be very clear what you use to compute your answer. If you use two subintervals in each direction, you get four subrectangles each with area 0.02. If you use the values of f given by the upper left corner of each subrectangle the approximation you get for the integral is 0.049. If for each subrectangle you use the average of the values at the corners, the approximation you get for the integral is 0.05395 (this is the two-variable version of a trapezoid sum).

5. There's a hard way to do this and an easy way. You should do it the easy way. 8π

6. The only way to do this is to sketch the region. The function being integrated has nothing to do with switching the order of integration.

7. 1/210

8. $\pi/2$

9. $4\pi/3$

10. 100 foot-pounds

11. $\frac{1}{4}(6\sqrt{37} + \operatorname{arcsinh} 6)$ Note: $\operatorname{arcsinh} x$ is the inverse hyperbolic sine function. The hyperbolic sine function is defined as $\sinh x = (e^x - e^{-x})/2$. Its inverse can be expressed using a logarithm as $\operatorname{arcsinh} x = \ln(x + \sqrt{x^2 + 1})$. There's also a hyperbolic cosine function: $\cosh x = (e^x + e^{-x})/2$. The hyperbolic trig functions have many properties in common with the regular, circular trig functions.