Math 14 **Exam 2** **Foote**

29 March 2000 100 Points

"Show enough work to justify your answers."

READ THIS !! Do any **five** problems. If you work on more than five, you will get credit for the best five. They are worth 20 points each.

You may use *Mathematica* to evaluate any integral, proper or improper, and any *finite* sum. When you use *Mathematica* as an essential part of solving a problem, in order to get full credit you **must** indicate in some detail how you use it, enough so it is clear to me how you draw your conclusions. If in doubt about how much to say, please ask. You do not need to indicate the use of *Mathematica* if you use it simply to check your work. If you have trouble with *Mathematica*, please ask.

Please begin each problem on an new sheet of paper. When you are done, put the pages in order with the problem number in the *upper right* corner, and staple them.

1. Evaluate $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$.

2. See what *Mathematica* gets for $\sum_{k=0}^{\infty}\frac{k+1}{k!}$. Carefully explain why its value is correct.

3. Consider $\sum_{k=2}^{\infty}\frac{k}{\sqrt{k^3-1}}$.
   a) State whether this series converges absolutely, converges conditionally, or diverges. (5 points, no partial credit)
   b) Carefully explain your conclusion in part a). (15 points)

4. Consider $\sum_{k=1}^{\infty}\frac{k}{e^k}=\sum_{k=1}^{\infty}ke^{-k}$.
   a) State whether this series converges absolutely, converges conditionally, or diverges. (5 points, no partial credit)
   b) Carefully explain your conclusion in part a). (15 points)

5. Consider $\sum_{k=1}^{\infty}a_k = a_1+a_2+a_3+\ldots$. Carefully explain what it means for this to converge conditionally and give an example with details.

6. Consider $\displaystyle\int_0^\infty \frac{1}{x+e^x}\,dx$.
   a) Carefully explain why this converges. (10 points)
   b) Find a value of b so that $\displaystyle\int_b^\infty \frac{1}{x+e^x}\,dx < .001$. Be clear why the inequality holds. (10 points)

7. Find the complete interval of convergence of $\displaystyle\sum_{k=1}^{\infty} \frac{(x-3)^k}{2^k\sqrt{k}}$, including the endpoints. Be sure to give complete explainations for the convergence or divergence at the endpoints.

8. Consider using a partial sum of a power series of $\cos x$ to approximate values of this function.
   a) State the power series of $\cos x$ expanded about $x = 0$. Give at least eight non-zero terms. (5 points)
   b) If $-\pi \le x \le \pi$, clearly indicate which terms of the series (underline them) can be used to approximate $\cos x$ with an error less than .002. Carefully state how you know the error will be this small. (10 points)
   c) Use the approximating sum you obtained in part b) to approximate $\cos 2$. (5 points)