Math 14 **Exam 1, Version A** **Foote**

14 February 2000 100 Points

"Show enough work to justify your answers."

When you use *Mathematica* as an essential part of solving a problem, in order to get full credit you **must** indicate in some detail how you use it, enough so it is clear to me how you draw your conclusions. If in doubt about how much to say, please ask. You do not need to indicate the use of *Mathematica* if you use it simply to check your work. If you have trouble with *Mathematica*, please ask.

1. Evaluate the following antiderivatives by hand, showing all steps. (15 points each)

   a) $\displaystyle\int x^2 \cos(3x)\,dx$ b) $\displaystyle\int \frac{3x^2-7}{(x-3)(x^2+1)}\,dx$ c) $\displaystyle\int \sec^4 x\ \tan^2 x\,dx$

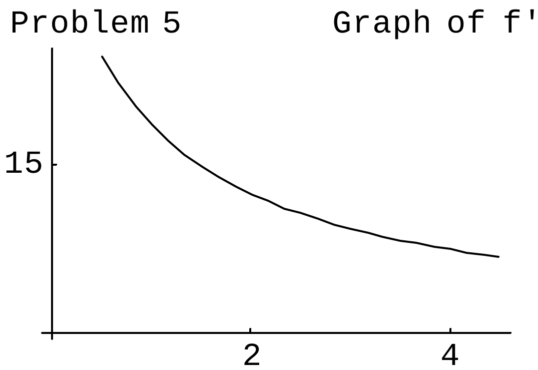
2. Consider $\int_0^2 x^3\,dx$.
   a) Approximate this integral using a trapezoid sum with four subintervals. Do this by hand, showing enough work so I can tell what you are thinking. Express your answer as a fraction or decimal. You may use a calculator or *Mathematica* to do the arithmetic, but you may **not** use the **trapezoid** function in Numerical-Integration.nb. (10 points)
   b) What is the error made by this approximation? Note: The question asks for the actual error, not an upper bound for the error. (5 points)

$$\left.\begin{matrix}|I-L_n|\\|I-R_n|\end{matrix}\right\}\le \frac{K_1(b-a)^2}{2n},\quad |I-T_n|\le\frac{K_2(b-a)^3}{12n^2},\quad |I-M_n|\le\frac{K_2(b-a)^3}{24n^2},\quad |I-S_n|\le\frac{K_4(b-a)^5}{180n^4}$$

3. Please reread the note about using *Mathematica* on the first page. In this problem you may **not** use *Mathematica*'s approximation for the integral, however you **may** use *Mathematica* to evaluate an approximating sum for the integral and to do arithmetic. Suppose you want to approximate the integral $\int_0^{\pi/2}\tan(\sin x)\,dx$ with an error less than .001.
   a) Determine how many subintervals are needed to do this for a midpoint sum, a trapezoid sum, or a Simpson's rule sum (choose one). Be clear which type of sum you are considering, and how you know the number of subintervals is adequate. (15 points)
   b) Based on part (a), what is your approximation for the integral? (5 points)

4. We spent a lot of time approximating integrals. Why? Given any integral, why not just find the antiderivative and plug in the endpoints and subtract, thereby getting the exact answer? After all, the exact answer would be better than an approximation. (5 points)

5. The graph of the derivative of a function is shown. (15 points)
   a) Write the approximations L_{10}, R_{10}, T_{10}, and M_{10} of $\int_2^4 f(x)\,dx$ in increasing order. Briefly say how you know.
   b) Which is closer to the exact value of $\int_2^4 f(x)\,dx$: L_{10} or R_{10}? Why?
   c) Give an upper bound for the error that the approximation L_{10} makes.



Answers and hints to selected questions.

1. See what *Mathematica* gets.
2. a) $T_4 = 17/4$ b) Error $= 1/4$
3. Numerical approximation of integral.
   a) Trapezoid: $n \geq 34$. Midpoint: $n \geq 24$. Simpson's: $n \geq 8$.
   b) $\int_0^{\pi/2} \tan(\sin x)\, dx \approx T_{34} = 1.33196$
5. a) $L_{10} < T_{10} < M_{10} < R_{10}$, b) R_{10} is closer.