Math 14 **Exam 3** **Name:** ______________________________

3 December 1999 100 Points

"Show enough work to justify your answers."

READ THIS !!! You are to work the problems 1 and 2, and then any four of the remaining problems.

When you use *Mathematica* as an essential part of solving a problem, indicate in some detail how you use it, enough so it is clear to me how you draw your conclusions. If in doubt about how much to say, please ask. You do not need to indicate the use of *Mathematica* if you use it simply to check your work. If you have trouble with *Mathematica*, please ask.

1. Evaluate the following by hand showing all steps. (10 points)
$$\int_0^2 \int_0^{2y} x\sqrt{y^3+1}\,dx\,dy$$

2. Suppose $w = f(x,y) = y2^{-x}$. Carefully draw three level curves: for $w = 1$, for $w = 0$, and the level curve passing through the point $(2,-2)$. Draw the curves all on the same coordinate system. Clearly label each curve with its value of w, and show the point $(2,-2)$. (10 points)

   READ THIS !!! Do any **four** of the remaining problems. If you work on more than four, you will get credit for the best four. (20 points each)

3. Sketch the region of integration and reverse the order of integration for
$$\int_1^{e^2} \int_{\ln x}^{2} \cos(xy^2)\,dy\,dx.$$

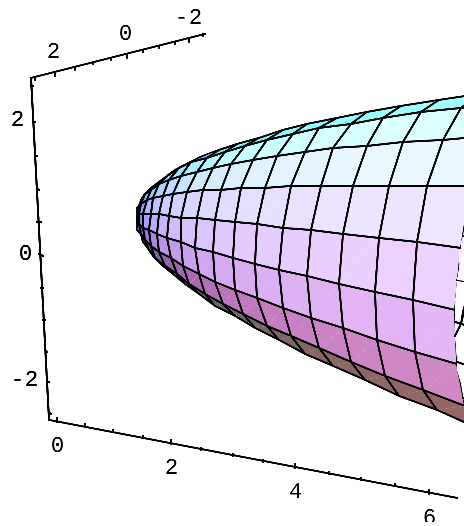
4. The surface $y = x^2 + z^2$ is pictured. In parts a) and b), set the appropriate variable to a constant, determine the resulting equation, sketch the corresponding curve on the surface, and say if it is a parabola, circle, ellipse, or hyperbola. Clearly label the curves you add to the picture.
   a) $x = 1$
   b) $y = 5$
   c) Find the coordinates of the points where these two curves intersect. Add the points to the picture and label with their coordinates.

5. The equation $xy^2z^3 = x^2 - yz^2 - 3$ defines some surface in three-dimensional space.
   a) Verify that the point $(1,2,-1)$ is on the surface and find an equation for the plane tangent to the surface at that point.
   b) Thinking of the equation as implicitly defining z as a function of x and y, compute $\frac{\partial z}{\partial x}$.

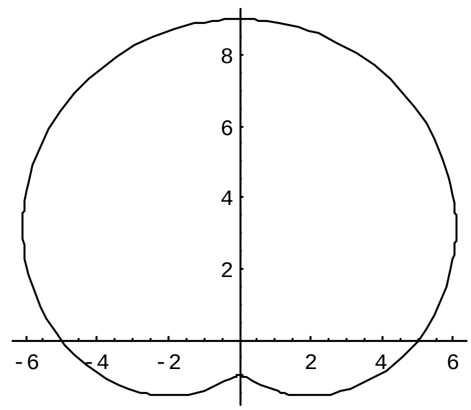
6. Let $f(x,y) = 3x^2+4y^3-12xy$. Find both stationary (critical) points of f and determine if each is a local minimum, local maximum, or a saddle point.

7. Find the maximum and minimum values of $f(x,y) = xy^2$ and their locations on the ellipse $x^2 + 2y^2 = 1$.

8. The graph of $r = 5 + 4\sin\theta$ is shown. Find the area it encloses.



Problem 4



Problem 8

Answers and hints for selected problems.

1. $104/9$

2. The curve passing through $(2,-2)$ is $y = -2^{x-1}$.

3. $\displaystyle\int_0^2 \int_1^{e^y} \cos(xy^2)\,dx\,dy$

4. a) parabola, b) circle, c) points of intersection: $(1,5,2)$ and $(1,5,-2)$

5. a) $6(x-1) + 3(y-2) - 8(z+1) = 0$

   b) $\dfrac{\partial z}{\partial x} = \dfrac{2x - y^2z^3}{3xy^2z^2 + 2yz}$

6. $(0,0)$ is a saddle point. $(9/4, 3/2)$ is a local minimum.

7. Six points need to be checked: $(1,0)$, $(-1,0)$, $(1/\sqrt{3}, 1/\sqrt{3})$, $(1/\sqrt{3}, -1/\sqrt{3})$, $(-1/\sqrt{3}, 1/\sqrt{3})$, and $(-1/\sqrt{3}, -1/\sqrt{3})$.
   Maximum value of $3^{-3/2}$ occurs at $(1/\sqrt{3}, 1/\sqrt{3})$ and $(1/\sqrt{3}, -1/\sqrt{3})$.
   Minimum value of $-3^{-3/2}$ occurs at $(-1/\sqrt{3}, 1/\sqrt{3})$ and $(-1/\sqrt{3}, -1/\sqrt{3})$.

8. 33π