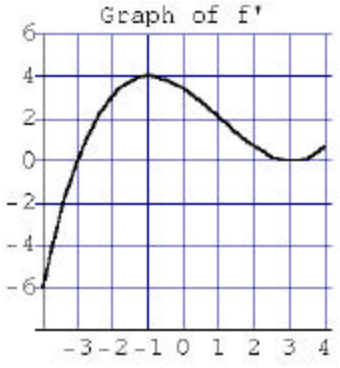
9 October 1998 100 Points

"Show enough work to justify your answers."

1. Evaluate $\lim_{x\to\infty} \dfrac{x^2-4x^3}{x^3+7}$. (10 points)

Read Carefully! Do any **six** of the following questions. If you do more than six, you will get credit for the best six. Fifteen points each.

2. The graph of the derivative of a function f is shown. Answer the following. Detailed reasons are not necessary.
   a) Where is f increasing?
   b) Where is f concave up?
   c) Where does f have stationary points?
   d) Where does f have local maxima?
   e) If $f(-1) = -2$, what is the tangent line to f at $x = -1$?



3. The graph of the derivative of a function f is shown.
   Explain why $f(1) - f(-3) > 7$.

4. Let $f(x) = \begin{cases} x^2\sin(1/x) + x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.
   It turns out that f is differentiable at $x = 0$, but *Mathematica* has trouble computing $f'(0)$. Using either a calculator or *Mathematica*, find an approximation for $f'(0)$. **Briefly describe what you do to find your approximation** by including some computation or picture that you found useful. Note: You can just use the first part of the formula to define the function in *Mathematica*.

5. If $f(3) = -2$ and $f'(x) < 1$ for all x, use the Racetrack Principle to say something about $f(8)$ and $f(0)$.

6. Sketch graphs of functions with the following properties. (Note that this problem has two parts.)
   a) $f(x)$ is defined for $-5 \le x \le 5$, $f(-5) = 0$, $f'(x) < 0$ for $-5 \le x \le -2$, $\lim_{x\to 0^-} f(x) = 3$, $f(0) = -1$, $f''(x) > 0$ for $x > 0$.
   b) $g(x)$ is continuous and defined for $x \ge 0$, $g(0) = 0$, $g''(x) > 0$ for $x > 3$, $\lim_{x\to\infty} g(x) = 2$.

7. State the definition of $f'(x_0)$ and use it to compute $f'(3)$ where $f(x) = x^2 + 2x$.

8. State the definition of $f'(x_0)$ and explain in words or pictures why it measures the slope of f at x_0.

9. We often use phrases such as "the maximum of f is 6" and "the maximum of f occurs at 6." These do not mean the same thing. Carefully explain the difference between these, in words or pictures.

10. Suppose $f(2) = 4$, $f'(2) = -1/2$, and $f''(x) < 0$ for $x > 0$. Explain why the graph of f must cross the x-axis somewhere to the left of $x = 10$. Hint: What can you say about $f(10)$?