Math 13 **Exam 3** **Foote**

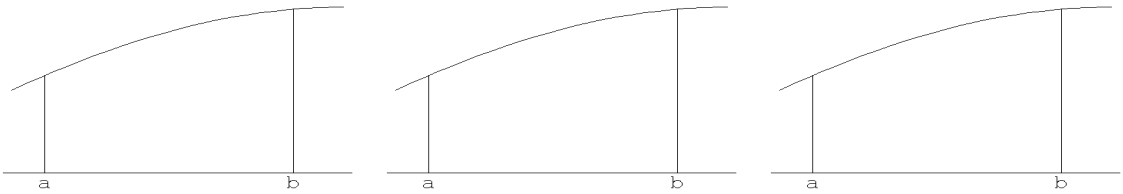
18 November 1998 100 Points

"Show enough work to justify your answers."

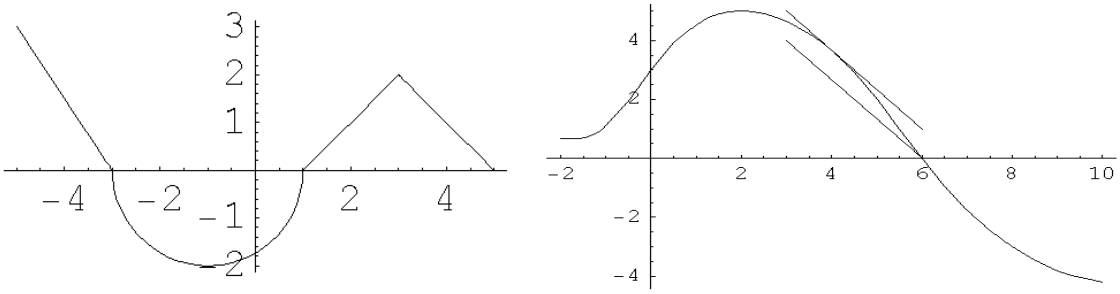
1. **Read Carefully!** Do any **two** of the following. If you do more than two, you will get credit for the best two. (10 points each)
   a) Evaluate $\int x^2 \sin(x^3)\,dx$. Show all steps.
   b) Compute the area between the curves $y = x^2$ and $y = x^3$.
   c) At some point the authors of the book claimed that $\int \ln x\,dx = x\ln x - x + C$. Verify this. Hint: What does a statement like this mean?

Read Carefully! Do any **four** of the remaining questions. If you do more than four, you will get credit for the best four. (20 points each)

2. Consider the integral $\int_a^b f(x)\,dx$, where three copies of the graph of f are shown.
   a) Assuming three subdivisions of the interval $[a, b]$ are used, draw the rectangles and trapezoids used in the left, right, and trapezoid approximating sums for the integral. Use different graphs for the different sums.
   b) Which of these sums overestimate and which underestimate the value of the integral? Which gives the closest estimate? Explain.



3. Let f be the function graphed below left.
   a) What is the value of $\int_3^{-1} f(x)\,dx$?
   b) Let $F(x) = \int_{-3}^{x} f(t)\,dt$. Find a formula for $F(x)$ for x in the interval $[1, 3]$.



4. Let f be the function graphed above right. Note that the line segment between $(3, 5)$ and $(6, 1)$ lies above the graph of f while the line segment between $(3, 4)$ and $(6, 0)$ lies below the graph of f. Use this information to find two numbers L and U such that $L < \int_3^6 f(x)\,dx < U$.

5. One of the basic properties of integrals is $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$.
   a) Suppose $a < b < c$. Using words and pictures explain why the formula is true.
   b) Explain why it is true even if $a < c < b$.

6. Let $G(x) = \int_0^x \sqrt{7+t^2}\,dt$.
   a) What does *Mathematica* get for $\int_0^3 \sqrt{7+x^2}\,dx$? Give both the exact value and the numerical approximation. (Note: The exact value involves a function we haven't studied.)
   b) Find the equation of the line tangent to the graph of G at $x = 3$. You may use numerical approximations.
   c) What is $G''(3)$ exactly?

7. Let f be the function graphed below. Let $F(x) = \int_{-3}^x f(t)\,dt$. Answer the following. Detailed reasons are not necessary.
   a) For what values of x in $[-4, 4]$ is $F(x)$ positive?
   b) Where is F increasing?
   c) Where is F concave down?
   d) Where does F have local maxima?