Math 13 **Exam 3** **Foote**

19 November 1997 100 Points

"Show enough work to justify your answers."

READ CAREFULLY! There are eight problems. Do any **five** of them. If you work on more than five you will get credit for the best five. I suggest reading through the entire exam before you start on any of the problems. (20 points each)

1. Let $G(x) = \int_{-2}^{x} f(t)\,dt$ and $H(x) = \int_{0}^{x} f(t)\,dt$ where f is the function graphed below.
   a) We have seen that G and H differ by a constant, that is, $G(x) = H(x) + C$, where C is some constant. Determine the exact value of C. (6 points)
   b) Find a formula for $H(x)$ for x in the interval $[1, 3]$. (7 points)
   c) At what values of x in $[-5, 5]$ does G have its local maxima and minima (include the endpoints). Briefly explain. (7 points)

2. Compute the shaded area (below) exactly. The curves are $y = \sin x$ and $y = \cos x$.

3. Let $f(x) = \int_{0}^{x} e^{-t^2}\,dt$ and $g(x) = x$. Use the Racetrack Principle to determine which is larger, $f(x)$ or $g(x)$ for $x \geq 0$.

4. In a science experiment a student collected the following data about a function f.

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	0.5	1.1	1.5	1.7	1.8

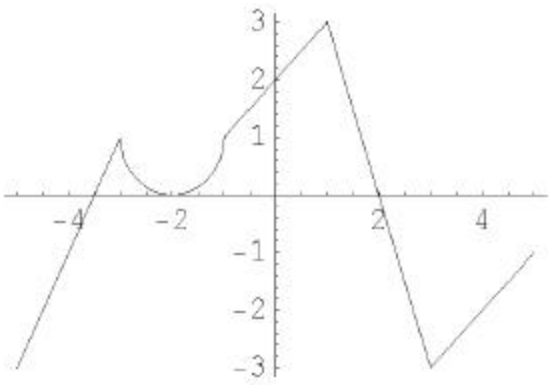
   a) Using this data, sketch a smooth curve in the space above that might represent the function. Assume that the data give a reasonable indication of how the function behaves, that is, no sudden jumps or twists. (5 points)
   b) Use the data to approximate $\int_{0}^{2} f(x)\,dx$. Specify which type of sum you are using (right sum, left sum, trapazoid sum, or midpoint sum) and show the appropriate rectangles or trapazoids in your graph. Write down enough of your computation so I can tell what you are doing. (10 points)
   c) Is the approximation you found in the previous part an under or over estimate of the true value of the integral? Explain. (5 points)

5. Evaluate the following antiderivatives. (10 points each)
   a) $\displaystyle\int (e^{2x} + \sec^2 5x)\,dx$ b) $\displaystyle\int (\cos 2x)\sqrt{5 + \sin 2x}\,dx$

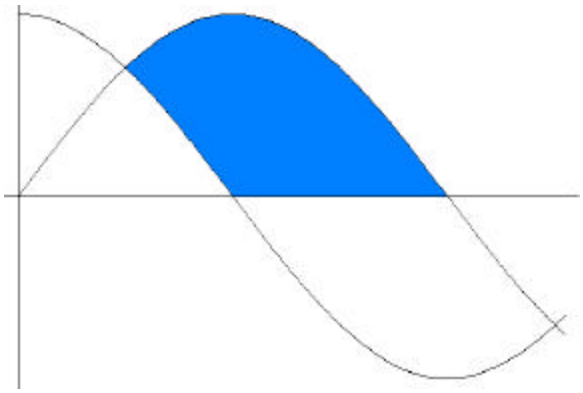
6. Let $F(x) = \int_{1}^{x} f(t)\,dt$, where the graph of f is shown below.
   a) Find the equation of the line tangent to the graph of F (not f!) at the point where $x = 4$. (10 points)
   b) Find the value of $F''(4)$. Does the graph of F lie above or below its tangent line near $x = 4$? Explain. (10 points)

7. The graph of f is the curve shown below. Use the fact that the graph lies between the two sets of lines to determine upper and lower bounds for $\int_{1}^{7} f(x)\,dx$.

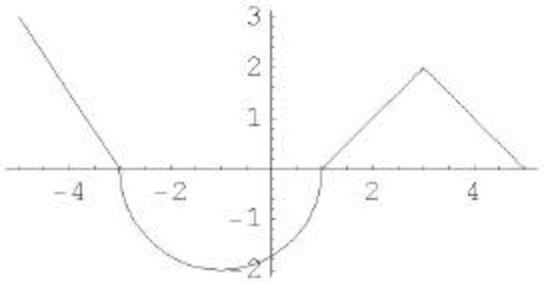
8. The pictured curve (below) is $y = \sqrt{x}$. Determine the value of b so that the amount of area shaded is 4 square units.



Problem 1



Problem 2



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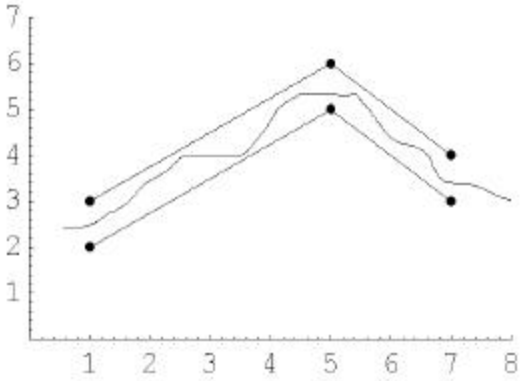
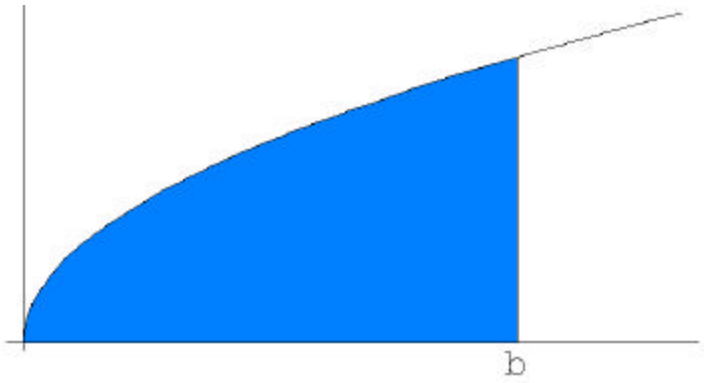


Problem 7



Problem 8