Math 13   **Exam 2**   **Foote**

1. Consider the graph of $y = g(x)$ shown (below).

   a) For what values of x is $g'(x)$ negative? How do you know? (7 points)
   b) For what values of x is $g''(x)$ positive? How do you know? (8 points)

2. Let $f(x) = 3x^2 - 2x$. Use the definition of the derivative (either form) to compute $f'(2)$. (10 points)

3. It was 12:00 midnight on a moonless night when the two Wallies finished packing The Bell into the back end of their van in the middle of the DePauw campus. "Let's get out of here!" one whispered to the other. They got into the van and drove through campus slowly so they wouldn't attract attention. Once they were off campus they drove at a normal speed through Greencastle until they reached city limits. "I think someone's following us!" the driver said, and he quickly sped up to 70 miles per hour, heading north on US-231. Unfortunately the car following them was a police car, which pulled them over. "You fellows are sure in a hurry," the cop said. It took about ten minutes for him to write them a speeding ticket, give them a stern lecture, and be sure they weren't drinking, after which he let them go. They drove the rest of the way to Crawfordsville at 55 miles per hour. They slowed down as they came into town, and one of them noticed it was 12:45a.m. "We're lucky the cop didn't want to look in the back of the van," he thought.

   Let $f(t)$ denote the distance in miles their car is from the DePauw campus, where t is measured in minutes after midnight. Sketch graphs of f and f' for $0 \le t \le 45$ that are compatible with the story and with each other. (It may be easier to think about f' first.) Clearly indicate the parts of both graphs related to the significant parts of the story: a) where they drove slowly through campus, b) where they drove through Greencastle, c) where they thought they were being followed, d) where they were stopped by the police, and e) where they arrived back in Crawfordsville. (20 points)

4. What is the derivative of $f(x) = 3x^{10} + \dfrac{2}{x^2}$? You may use any method you wish. (10 points)

5. The graph of the derivative of f is shown below.

   a) For what value of x is $f(x)$ the smallest? Briefly explain. (5 points)
   b) Assuming $f(1) = 3$, what is the formula $\ell(x)$ of the line tangent to the graph of f at $x = 1$? (5 points)
   c) Put the following six numbers in order. Briefly explain. (10 points)
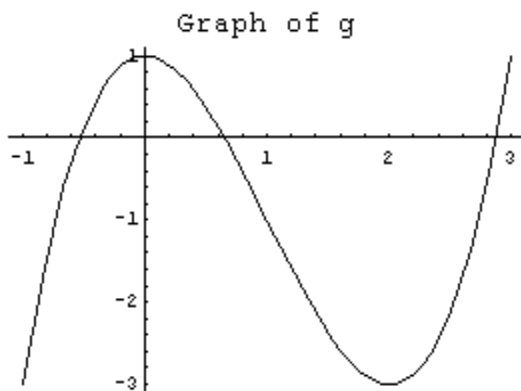 $$f(-1), \qquad f(1), \qquad f(3), \qquad \ell(-1), \qquad \ell(1), \qquad \ell(3)$$

6. The following table gives some values of a function f.

   a) Based on this, give an approximation of $f'(1.0)$. Use your calculator or *Mathematica* to do the arithmetic, but write down enough of your computation so I can see where your answer comes from. (10 points)
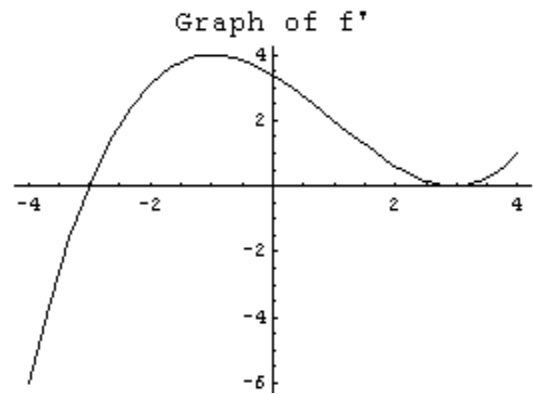
x	$f(x)$
0.8	0.717
0.9	0.783
1.0	0.841
1.1	0.891
1.2	0.932

   b) **Extra Credit.** Give an approximation of $f''(1.0)$. (10 points)

7. Suppose $f(3) = 5$ and $f'(x) \leq 2$ for all x. Based on the Racetrack Principle, what can you conclude about $f(6)$ and $f(0)$? (15 points)

Graph of g

Graph of f'

Problem 1

Problem 5