Math 13 **Exam 1** **Foote**

22 September 1995 100 Points

"Show enough work to justify your answers."

1. The graph of $f(x) = A\,b^x$ is shown (below), including two points on the graph, $(-1, 15/4)$ and $(1, 5/3)$. Find the exact values of A and b. (15 points)

2. Let $f(x) = \begin{cases} -x & \text{if } x \le 0 \\ x-1 & \text{if } x \ge 0 \end{cases}$ for $-2 \le x \le 2$. (5 points each)

   a) Explain carefully why this is *not* a properly defined function. Make a slight change in the definition of f so that it will be a function. Be sure your change is clear. The function must still be piecewise defined.
   b) Graph your function.
   c) What is the range of your function?

3. It was 12:00 noon as Wally Wabash pulled onto the west-bound ramp of Interstate 74 just outside of Indianapolis. As he quickly accelerated from 40 mph to 80 mph he said to himself "I'm really going to have to push it to get to Dr. Foote's 1:10 calculus class!" After about 15 minutes of cruising at a steady speed his radar detector went off. Wally quickly slowed down to a legal speed, his heart pounding as he went by the police car parked just off the edge of the road. After a few more minutes he felt he was safely out of danger, and he gradually picked up his speed again. Finally he came to the Crawfordsville exit. Slowing down to 40 mph, he pulled onto the exit ramp, and then came to a stop at the STOP sign at State Road 32 just east of Crawfordsville. Glancing at his watch he noted it was 12:45. "Time to spare!" he thought. To be continued ...

   Let $f(t)$ denote the distance in miles from the point where Wally got onto I–74, where t is measured in minutes after noon. Sketch graphs of f and f' for $0 \le t \le 45$ that are compatible with the story and with each other. (It may be easier to think about f' first.) Clearly indicate the parts of your graphs related to the significant parts of the story, in particular, where he is speeding up and braking, where the radar detector goes off, and where he comes to a stop. (15 points)

4. The graph of f' is shown below. Assume that $f(0) = 5$.
   (5 points each)

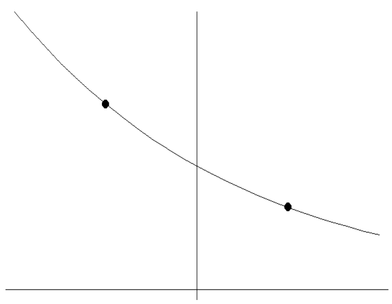
   a) For what values of x is f increasing? How do you know?
   b) For what values of x is f concave up? How do you know?
   c) For what values of x is $f''(x) = -1$? How do you know?
   d) What is the equation of the line tangent to the graph of f at $x = 0$?
   e) Explain why $f(3) > 8$. (For 5 points extra credit, explain why $f(3) > 10$.)

5. Fill in the blanks with '$>$' or '$<$'. Explain either in terms of steepness of graphs or speeds of cars. (15 points)

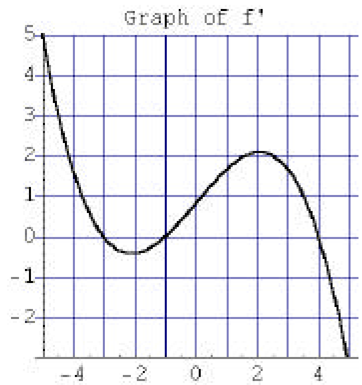
   If $f'(x) < g'(x)$ for all x and $f(-3) = g(-3)$, then

   a) $f(x)$____$g(x)$ for $x > -3$, and (b) $f(x)$____$g(x)$ for $x < -3$.

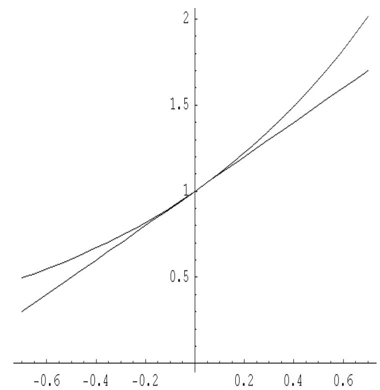
6. It turns out that $f(x) = e^x$ can be approximated by $\ell(x) = x + 1$ for x near 0. (This is because ℓ is the tangent line of f at $x = 0$.) The graphs of f and ℓ are shown. Based on the graphs, use your calculator to estimate the maximum error made by this approximation for $-0.2 \leq x \leq 0.2$. Explain. (Remember that the error can be measured by $|f(x) - \ell(x)|$.) (15 points)



Problem 1



Problem 4



Problem 6