**Math 112** **Exam 3** **Name:**

18 April 2007 100 Points

No *Mathematica*

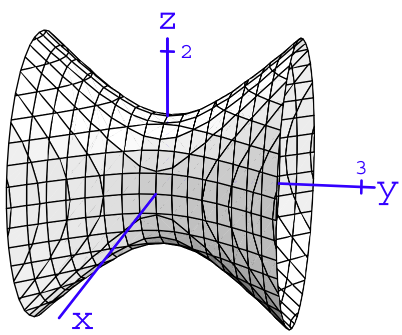
*"Show enough work to justify your answers."*

**READ THIS !!** There are two parts. In Part I you are to do every problem. In Part II, you have a choice of problems.

**Part I.** Do every problem. Do your work on the exam. (10 points each)

1. A portion of the surface $x^2-y^2+z^2=1$ is shown. For each part, determine the resulting equation when the surface is sliced by the given plane, and sketch the corresponding curve on the surface. Label each curve with (a), (b), or (c), as appropriate.

   (a) Sliced by the plane $y=-2$ (b) Sliced by $x=1$ (c) Sliced by $z=0$




2. Compute the plane tangent to the surface $x^2+2xy=1-y^2/z$ at the point $(-2,3,1)$. Leave the equation in a form that exhibits the point of tangency.

3. Let $z=f(x,y)=x^2+4y^2$. Draw three level curves of $f$: the one for which $z=16$, the one for which $z=4$, and the one passing through the point $(2,1)$. Label each curve with its $z$-value.

4. Evaluate: $\displaystyle\int_1^2\int_1^{x^2}\left(x^2+\frac{2y}{x^2}\right)dy\,dx$

**Part II. Read Carefully!** Do any **four** of the remaining problems. If you work on more than four, you will get credit for the best four. They are worth 15 points each. I suggest quickly reading though them to see what the problems are.

Do the work for these on blank paper. **Please** begin each problem on a new sheet of paper. Avoid writing in the upper left corner of the page (where the staple will go). When you are done, put the pages in order with the problem number in the *upper right* corner, and staple them to the exam.

5. Laplace's equation in two variables is $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. Determine if $f(x, y) = \ln(x^2 + y^2)$ satisfies Laplace's equation.

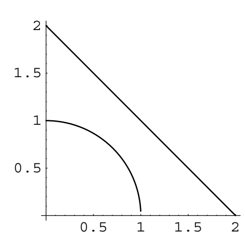
6. Verify that the point $(1, 3, 2)$ is on the surface $x^4 + 4 = (y^2 - z^2)x^3$. Find an approximation of the $x$-coordinate of the nearby point $(x, 3.1, 1.8)$ on the surface.

7. Find the stationary points of $f(x, y) = y^2 + 2xy + \frac{1}{3}x^3 - 3x$. For each stationary point, determine if it is a local maximum, local minimum, or saddle point.

8. Consider the function $f(x, y) = 2y - 3x + 5$ and the ellipse $x^2 + 2y^2 = 44$. Answer the following (with full reasoning): The maximum value of $f$ on the ellipse is _____, and the minimum value of $f$ on the ellipse is _____.

9. Sketch the region of integration and reverse the order of integration. Do not evaluate.
$$\int_{-2}^{0} \int_{y^2}^{4} (\sin x + \cos y)\, dx\, dy$$

10. Express the following integral in polar coordinates, where $R$ is the region in the first quadrant below the line $x + y = 2$ and outside the circle $x^2 + y^2 = 1$, shown below. Do not evaluate. Your answer should be simplified and "ready to evaluate."
$$\iint_R \frac{x}{x^2 + y^2}\, dA$$



Selected answers and hints.

1. The equations in (a), (b), and (c) tell you a lot about where the curve will be on the surface.

4. See what *Mathematica* gets.

6. $x \approx .873$

7. There are two stationary points. One is a local minimum, the other is a saddle.

8. The maximum and minimum values are 27 and $-17$, respectively. Here are two strategies to solve the equations:

    - Divide the first equation by the second, eliminating $\lambda$, to get an equation relating $x$ and $y$. Plug this into the third equation.
    - Solve the first equation for $x$ and the second for $y$ in terms of $\lambda$. Plug these into the third equation and solve for $1/\lambda$ to get $1/\lambda = \pm 4$.

9. $\displaystyle\int_0^4 \int_{-\sqrt{x}}^0 (\sin x + \cos y)\, dy\, dx$

10. Show that the equation of the line in polar coordinates is $r = \dfrac{2}{\cos\theta + \sin\theta}$.