Math 112 **Exam 2** **Name:**

14 March 2007 100 Points

"Show enough work to justify your answers."

READ THIS !! Do any **five** problems. If you work on more than five, you will get credit for the best five. They are worth 20 points each. I suggest quickly reading though the exam to see what the problems are.

You may use *Mathematica* on any problem to help you think, however, you may **not** use it as as part of a solution except as noted. If you have trouble with *Mathematica*, please ask.

Please begin each problem on a new sheet of paper. Avoid writing in the upper left corner of the page (where the staple will go). When you are done, put the pages in order with the problem number in the *upper right* corner, and staple them to this page.

1. Explain in your own words the difference between an infinite sequence and an infinite series. Explain what convergence means for each. You may refer to specific examples if you like.

2. Explain why the following series converges and determine its sum.
$\displaystyle\sum_{k=0}^{\infty} \frac{3+(-1)^k}{3^k}$.

3. Consider the series $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{\sqrt{k}+k^6}$. It can be shown to converge by comparing it to $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^4}$.

   (a) How many terms of the original series are needed in order to approximate its sum with an error less than .001? Carefully indicate how you know the error will be this small. Your answer may not depend on the use of *Mathematica* to evaluate the entire series. You may, however, use *Mathematica* to do arithmetic. (15 points)

   (b) Use *Mathematica* to compute a decimal approximation for the sum of the original series based on part (a). Don't simply give the approximation, but indicate what you do to compute it. (5 points)

4. Give an example of a conditionally convergent series other than the alternating harmonic series. Give a full explanation of why you know it converges conditionally.

5. We have that $\arctan x = \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$ for all values of x that make the series converge.

   (a) Determine the full interval of convergence for this power series, including endpoints. (15 points)

   (b) Use $x = 1$ to get an infinite series equal to π. (5 points)

6. Consider the series $\sum_{k=0}^{\infty}(-1)^k \frac{1}{3^k - k}$.

   (a) State whether this series converges absolutely, converges conditionally, or diverges. (5 points. No work necessary. No partial credit.)

   (b) Carefully justify your claim in part (a). (15 points)

7. Define the function f by $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \ldots$ for all values of x for which the series converges. The goal of this problem is to find a closed-form (non-series) formula for $f(x)$.

   (a) Suppose that F is an antiderivative for f. Find a power series for $F(x)$. (7 points)

   (b) Use part (a) to find a closed-form formula for $F(x)$. (7 points)

   (c) Use part (b) to find a closed-form formula for $f(x)$. (6 points)

8. Consider using a Taylor polynomial for $\sin x$ to approximate values of $\sin x$.

   (a) State the power series of $\sin x$ expanded about $x = 0$. Give at least eight terms. (5 points)

   (b) If $-2 \leq x \leq 2$, clearly indicate which terms of the series (underline them) should be used to approximate $\sin x$ with an error less than .001. Carefully state how you know the error will be this small. You may use *Mathematica* to do arithmetic. (10 points)

   (c) Use *Mathematica* and the approximating sum you obtained in part (b) to approximate $\sin(1.5)$. Don't simply give the approximation, but indicate what you do to compute it. (5 points)

Selected answers and hints.

2. Look at some of the problems in Section 11.2 and think about the tools you had at that point. There are *very few* series for which it is reasonable to ask for the exact value of the sum. The fact that you are asked for the sum narrows the type of series it can be considerably.

3. (a) 7 term are needed.

   (b) .580157

5. (a) $-1 \leq x \leq 1$

   (b) You need to know what $\arctan 1$ is! This is an angle whose tangent is 1.

6. (a) Intuition: This is like a geometric series with ratio $-1/3$, so it should converge absolutely.

   (b) The AST is a waste of time on this one (why?). You can't use the comparison test directly (why not?). Considering $\sum \frac{1}{3^k - k}$, you can't compare it with $\sum \frac{1}{3^k}$ (why not?). What can you compare it with? The Ratio Test is also a good choice because the series is nearly geometric.

7. (c) $1/(1-x)^2$

8. (b) You can use either the AST error estimate or Taylor's Theorem. The terms through $x^9/9!$ are needed.

   (c) 0.997497