Math 112 **Exam 1** **Name:**

6 February 2007 100 Points

"Show enough work to justify your answers."

Read carefully: This exam has two parts. You are to do **all** of the problems on Part I. On Part II you will have a choice of problems.

Part I. Do all of the problems in this part. (25 points total)

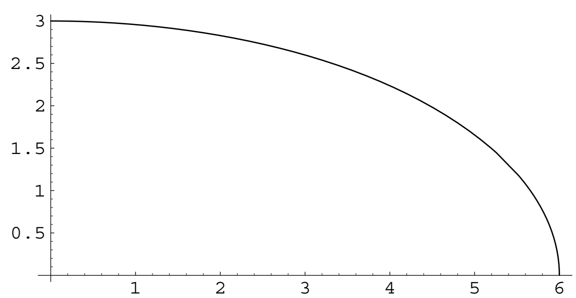
1. Evaluate (10 points): $\displaystyle\int \frac{x}{3x^2+5}\,dx$

2. Evaluate (10 points): $\displaystyle\int xe^{2x}\,dx$

3. Evaluate (5 points): $\displaystyle\lim_{x\to 0} \frac{\tan 2x}{x}$

Part II. Do **any five** of the remaining problems. If you work on more than five, you will get credit for the best five. Suggestion: Read them all quickly to see what they are like. (15 points each; 75 points total)

4. A portion of the graph of $f(x) = \frac{1}{2}\sqrt{36-x^2}$ is shown. Consider $\int_2^5 f(x)\,dx$.



   (a) Draw the picture that illustrates the approximating sum T_4 for the integral.

   (b) Compute T_4. You may use a calculator to do the arithmetic. Show enough detail so I can tell what you are thinking.

   (c) Does T_4 overestimate or underestimate the exact value of the integral? What property of the graph causes this?

5. Evaluate: $\displaystyle\int \sec^6 x\, dx$

6. Evaluate the following.

$$\int \frac{1}{x\sqrt{4-x^2}}\, dx$$

7. The base of a solid is the region in the first quadrant inside $y = x^2$ and below $y = 4$. Cross sections of the solid perpendicular to the y-axis are squares. Find the volume of the solid.

8. Compute the length of the graph of $f(x) = \frac{2}{3}x^{3/2}$ between $x = 0$ and $x = 8$.

9. Evaluate: $\displaystyle\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^x$

10. Explain why the following improper integral converges by comparing it with another improper integral. Give full reasoning, including an appropriate inequality and a reason for the convergence of the integral you use for comparison.

$$\int_3^\infty \frac{1}{2x^4 + 5x + 7}\, dx$$

11. Suppose that $f(x) > 0$ for $x \geq 1$ and that $\int_1^\infty f(x)\, dx$ converges. Explain in words, symbols, or pictures, why $\int_b^\infty f(x)\, dx$ is the error made by the approximation $\int_1^b f(x)\, dx \approx \int_1^\infty f(x)\, dx$, and why the error can be made small by taking b to be large.

Selected answers and hints.

1. For this and other antiderivatives, see what *Mathematica* gets.

3. Be sure the use of L'Hôpital's Rule is warranted.

4. (b) $T_4 = 7.10$

6. Note: When you *change* variables, you need to use a *different* variable. For example, the substitution can't be $x = 2\sin x$.

7. 8

8. 52/3

9. e^3