Math 112       Exam 1       Name:

6 February 2007       100 Points

“Show enough work to justify your answers.”

Read carefully: This exam has two parts. You are to do all of the problems on Part I. On Part II you will have a choice of problems.

Part I. Do all of the problems in this part. (25 points total)

1. Evaluate (10 points): \[ \int \frac{x}{3x^2 + 5} \, dx \]

2. Evaluate (10 points): \[ \int xe^{2x} \, dx \]

3. Evaluate (5 points): \[ \lim_{x \to 0} \frac{\tan 2x}{x} \]

Part II. Do any five of the remaining problems. If you work on more than five, you will get credit for the best five. Suggestion: Read them all quickly to see what they are like. (15 points each; 75 points total)

4. A portion of the graph of \( f(x) = \frac{1}{2} \sqrt{36 - x^2} \) is shown. Consider \( \int_{2}^{5} f(x) \, dx \).

(a) Draw the picture that illustrates the approximating sum \( T_4 \) for the integral.

(b) Compute \( T_4 \). You may use a calculator to do the arithmetic. Show enough detail so I can tell what you are thinking.

(c) Does \( T_4 \) overestimate or underestimate the exact value of the integral? What property of the graph causes this?
5. Evaluate: \[ \int \sec^6 x \, dx \]

6. Evaluate the following.
\[ \int \frac{1}{x \sqrt{4 - x^2}} \, dx \]

7. The base of a solid is the region in the first quadrant inside \( y = x^2 \) and below \( y = 4 \). Cross sections of the solid perpendicular to the \( y \)-axis are squares. Find the volume of the solid.

8. Compute the length of the graph of \( f(x) = \frac{2}{3}x^{3/2} \) between \( x = 0 \) and \( x = 8 \).

9. Evaluate: \( \lim_{x \to \infty} \left( 1 + \frac{3}{x} \right)^x \)

10. Explain why the following improper integral converges by comparing it with another improper integral. Give full reasoning, including an appropriate inequality and a reason for the convergence of the integral you use for comparison.
\[ \int_{3}^{\infty} \frac{1}{2x^4 + 5x + 7} \, dx \]

11. Suppose that \( f(x) > 0 \) for \( x \geq 1 \) and that \( \int_{1}^{\infty} f(x) \, dx \) converges. Explain in words, symbols, or pictures, why \( \int_{1}^{b} f(x) \, dx \) is the error made by the approximation \( f_{1}^{b} f(x) \, dx \approx f_{1}^{\infty} f(x) \, dx \), and why the error can be made small by taking \( b \) to be large.
Selected answers and hints.

1. For this and other antiderivatives, see what *Mathematica* gets.

3. Be sure the use of L'Hôpital's Rule is warranted.

4. (b) \( T_4 = 7.10 \)

6. Note: When you *change* variables, you need to use a *different* variable. For example, the substitution can’t be \( x = 2\sin x \).

7. 8

8. 52/3

9. \( e^3 \)