Math 112 **Exam 3** **Name:**

24 April 2006 100 Points

"Show enough work to justify your answers."

READ THIS !!! Do any **five** problems. If you work on more than five, you will get credit for the best five. (20 points each)

Please begin each problem on an new sheet of paper. Avoid writing in the upper left corner of the page (where the staple will go). When you are done, put the pages in order with the problem number in the *upper right* corner, and staple them to the exam.

You may use *Mathematica* as an essential part of solving only the last problem. You may use it on other problems to check your work, and you do not need to indicate that use. If you have trouble with *Mathematica*, please ask.

1. Let $z = f(x, y) = xy$.

   (a) Carefully draw three level curves in quadrants I and II: for $z = 2$, for $z = 0$, and the level curve passing through the point $(-2, 2)$. Draw the curves all on the same coordinate system. Clearly label each curve with its value of z, and show the point $(-2, 2)$. Size your sketch to be compatible with the information to be shown (not too crammed together).

   (b) Compute the line tangent to the level curve of f where it passes through $(-2, 2)$. Add the line to the sketch.

2. Suppose that the variables u, v, and w are related by the equation $u^2 - v + w^3 = \sin(uv)$.

   (a) Compute an equation that relates du, dv, and dw. (8 points)

   (b) Thinking of u as a function of v and w, use the formula in part a) to find $\frac{\partial u}{\partial v}$ and $\frac{\partial u}{\partial w}$ in terms of u, v, and w. (12 points)

3. Let $f(x, y) = 3x^2 - 4y^3 + 12xy$. Find both stationary (critical) points of f and determine if each is a local minimum, local maximum, or a saddle point.

4. Find the maximum and minimum values of $f(x, y) = 2x - y$ on the disk $x^2 + y^2 \leq 20$. (Note: The region is the circle and its interior.)

5. Find the volume under the plane $3x + 2y + z = 10$ over the rectangle with opposite corners $(-1, -3)$ and $(2, 1)$.

6. Sketch the region of integration and reverse the order of integration. Evaluate either integral.
$$\int_0^1 \int_y^1 e^{-x^2}\, dx\, dy.$$

7. Find the volume of the solid above the (x, y)-plane that is bounded by the paraboloid $z = 4 - x^2 - y^2$. There are two ways to do this. One is to note that horizontal slices are circles. The other is to use a double integral. If you use a double integral, it is easier to evaluate in polar coordinates.

8. The polar curve $r = \cos 2\theta$ is shown. Find the area of one loop. You may use *Mathematica* to evaluate your integral. If you do, be clear on your exam both your integral and the *Mathematica* result. If you do it by hand, you will need the identity $\cos^2 \phi = \frac{1}{2}(1 + \cos 2\phi)$.

Selected answers and hints.

1. (b) $2(x + 2) - 2(y - 2) = 0$
2. (b) $\dfrac{\partial u}{\partial v} = \dfrac{\cos(uv) + 1}{2u - \cos(uv)v}$
3. It's a good idea to check your critical points by plugging their coordinates into the equations they are supposed to satisfy. $(4, -2)$ is a local minimum. The other critical point is a saddle point.
4. There are no interior critical points, and so the max and min occur on the boundary. There are two critical points on the boundary. The maximum and minimum values are 10 and -10.
5. 126
6. One of the integrals can't be evaluated directly. The common value is $\dfrac{1}{2} - \dfrac{1}{2e}$.
7. 8π
8. $\pi/8$