Math 112 **Exam 2** **Name:**

17 March 2006 100 Points

"Show enough work to justify your answers."

READ THIS !! Do any **five** problems. If you work on more than five, you will get credit for the best five. They are worth 20 points each.

You may use *Mathematica* as convenient except as noted. When you use *Mathematica* as an essential part of solving a problem, in order to get full credit you **must** indicate in some detail how you use it, enough so it is clear to me how you draw your conclusions. If in doubt about how much to say, please ask. You do not need to indicate the use of *Mathematica* if you use it simply to check your work. If you have trouble with *Mathematica*, please ask.

Please begin each problem on a new sheet of paper. Avoid writing in the upper left corner of the page (where the staple will go). When you are done, put the pages in order with the problem number in the *upper right* corner, and staple them to this page.

1. Suppose that $\sum_{k=0}^{\infty} a_k(x+2)^k$ converges when $x = 0$ and diverges when $x = 6$. For each of the following, say if the statement *must* be true, *might* be true, or *cannot* be true. Briefly explain. (5 points each)
   (a) The series converges when $x = 8$.
   (b) The series converges when $x = 3$.
   (c) The series converges when $x = -3$.
   (d) The series converges when $x = -4$.

2. Geometric series.
   (a) Explain what a geometric series is. Your description should be general, that is, you should not refer to a specific example. (8 points)
   (b) Give two examples of geometric series, one that converges and one that diverges. Give the sum of the convergent one. (12 points)

3. Consider the series $\sum_{k=0}^{\infty} \frac{3^k}{\sqrt{k} + 6^k}$. It can be shown to converge by comparing it to $\sum_{k=0}^{\infty} \frac{1}{2^k}$.
   (a) How many terms of the original series are needed in order to approximate its sum with an error less than .001? Carefully indicate how you know the error will be this small. Your answer may not depend on the use of *Mathematica* to evaluate the entire series. You may, however, use *Mathematica* to do arithmetic. (15 points)
   (b) Compute a decimal approximation for the sum of the original series based on part (a). Don't simply give the approximation, but indicate what you do to compute it. (5 points)

4. Determine the full interval of convergence for the following power series, including endpoints: $\sum_{k=1}^{\infty} \frac{\sqrt{k}(x+2)^k}{3^k}$

5. Consider the series $\sum_{k=2}^{\infty} (-1)^k \frac{1}{k \ln k}$.

   (a) State whether this series converges absolutely, converges conditionally, or diverges. (5 points, no partial credit)

   (b) Carefully justify your claim in part (a). (15 points)

6. Consider the series $\sum_{k=1}^{\infty} (-1)^k \frac{5^k}{k!}$.

   (a) State whether this series converges absolutely, converges conditionally, or diverges. (5 points, no partial credit)

   (b) Carefully justify your claim in part (a). (15 points)

7. The goal of this problem is to find a power series for $\arctan x$. Your answer must be independent of *Mathematica*.

   (a) Write a power series for $\frac{1}{1+x^2}$. Briefly indicate how you know you have an appropriate power series. Write your answer in summation notation or give at least six non-zero terms.

   (b) Use the result of part (a) to find a power series for $\arctan x$. (Recall how $\arctan x$ is related to $1/(1+x^2)$.) Write your answer in summation notation or give at least six non-zero terms.

8. Consider using a Taylor polynomial for e^x to approximate values of e^x.

   (a) State the power series of e^x expanded about $x = 0$. Give at least eight terms. (5 points)

   (b) If $-2 \leq x \leq 2$, clearly indicate which terms of the series (underline them) can be used to approximate e^x with an error less than .001. Carefully state how you know the error will be this small (use Taylor's Theorem). (10 points)

   (c) Use the approximating sum you obtained in part (b) to approximate $e^{1.5}$. Don't simply give the approximation, but indicate what you do to compute it. (5 points)

Selected answers and hints.

1. Determine what the interval of convergence might be.

3. (a) You need 11 terms ($n = 10$).
   (b) 1.91704

4. $-5 < x < 1$

5. The series converges conditionally. To show it converges, use the AST. To so that it does not converge absolutely, use the integral test. Note: You can't prove absolute convergence with the AST.

7. Antidifferentiate the power series for $1/(1 + x^2)$.

8. (b) You need 11 terms ($n = 10$).
   (c) 4.48169