Math 112 **Exam 1** **Name:**

8 February 2006 95 Points

"Show enough work to justify your answers."

1. Evaluate **any three** of the following antiderivatives. If you work on more than three, you will get credit for the best three. Continue on the back of this page if you need more space. Do all work by hand, showing all steps. (12 points each)

   a) $\int \sqrt{x} \ln x \, dx$ b) $\int e^{\sqrt{x}} \, dx$ (let $x = u^2$)

   c) $\int \frac{1}{\sqrt{9 - x^2}} \, dx$ d) $\int \sec^4 x \, dx$ e) $\int x\sqrt{x^2 + 16} \, dx$

$$|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2} \qquad |I - M_n| \leq \frac{K_2(b-a)^3}{24n^2} \qquad |I - S_n| \leq \frac{K_4(b-a)^5}{180n^4}$$

2. Give short answers. (15 points) In the midpoint error estimate formula,
   (a) What do I and M_n represent? (4 points)
   (b) Which part of the formula represents the exact value of the error? (3 points)
   (c) How does the formula imply that under certain circumstances the error is small, and what are those circumstances? (4 points)
   (d) Why does the corresponding error formula for Simpson's Rule generally lead us to expect that Simpson's rule will give a better approximation than the midpoint sum? (4 points)

3. You need to use *Mathematica* in this problem. To get full credit, you **must** clearly indicate what you use it for, how you use it, and the conclusions you draw from it. If you have trouble with *Mathematica*, ask for help. (14 points)

   Consider $\int_1^6 \frac{x}{x + \sin x} \, dx$.
   (a) Determine the number of subintervals needed for the midpoint sum error estimate to guarantee that the midpoint sum approximation of the integral is good to two decimal places, that is, the error should be less than .005. Clearly indicate how you arrive at your value for K_2. (10 points)
   (b) What is the approximate value of the integral based on part (a)? (4 points)

4. Consider $\int_1^2 \frac{1}{\sqrt{x-1}} \, dx$. Explain why this integral is improper. Then express it as a limit and evaluate it, doing all work by hand. (10 points)

5. Evaluate the following limits. If you use L'Hôpital's Rule, indicate why it is okay to use it. (10 points each)

   (a) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$ (b) $\lim_{x \to 1} \frac{\ln x}{x^2 - x}$

Selected answers and hints.

1. Remember to check your answers by differentiating them.
2. (b) The answer is *not* $\dfrac{K_2(b-a)^3}{24n^2}$.

   (d) The answer is *not* because 180 is bigger than 24.
3. (a) A reasonable value for K_2 is 0.4. In this case the best value for n is 21.

   (b) $M_{21} = 4.93266$
4. See what *Mathematica* gets.
5. (a) 0 (b) 1