Math 112 (Foote) **Exam 2**

26 March 2001 100 Points

"Show enough work to justify your answers."

READ THIS !! Do any **five** problems. If you work on more than five, you will get credit for the best five. They are worth 20 points each.

You may use *Mathematica* as convenient except as noted. When you use *Mathematica* as an essential part of solving a problem, in order to get full credit you **must** indicate in some detail how you use it, enough so it is clear to me how you draw your conclusions. If in doubt about how much to say, please ask. You do not need to indicate the use of *Mathematica* if you use it simply to check your work. If you have trouble with *Mathematica*, please ask.

1. When the terms of a series go to zero, it is a common mistake to conclude that the series has to converge. Give two examples of divergent series for which the terms go to zero. Explain why they are divergent.

2. Consider the series $\sum_{k=0}^{\infty} \frac{2^k}{k^2+3^k}$. It can be shown to converge by comparing it to $\sum_{k=0}^{\infty} \frac{2^k}{3^k}$.

   (a) Which terms of the original series are needed in order to approximate its sum with an error less than .001? Your answer may not depend on the use of *Mathematica* to evaluate the entire series. You may, however, use *Mathematica* to do arithmetic. (15 points)

   (b) Compute a decimal approximation for the sum of the original series based on part a). Don't simply give the approximation, but indicate what you use to compute it. (5 points)

3. Suppose that $\sum_{k=0}^{\infty} a_k(x-3)^k$ converges when $x=1$ and diverges when $x=7$. For each of the following, say if the statement *must* be true, *might* be true, or *cannot* be true. Briefly explain. (5 points each)

   (a) The series converges when $x=2$.

   (b) The series converges when $x=5$.

   (c) The series converges when $x=8$.

4. Determine all values of x for which the series $\displaystyle\sum_{k=1}^{\infty} \frac{(x-6)^k}{\sqrt{k}}$ converges.

5. Consider the series $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{k}{k^3+1}$.

   (a) State whether this series converges absolutely, converges conditionally, or diverges. (5 points, no partial credit)

   (b) Carefully justify your claim in part a). (15 points)

6. Give a clear definition of what it means for a series $\displaystyle\sum_{k=1}^{\infty} a_k$ to converge conditionally. (Your definition should apply to series in general and not to some specific series.) After your definition give an example of a specific series that converges conditionally and explain.

7. Let $f(x) = \sec x$. Give the 9^{th}degree Maclaurin polynomial for f.

8. Let $w = xz^3 + x^2 \cos y + \dfrac{y^2}{z}$. Compute $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$, $\frac{\partial w}{\partial z}$, and dw. Show all work (do not use *Mathematica*).