Math 112 **Exam 1** **Foote**

12 February 2001 100 Points

"Show enough work to justify your answers."

1. Do any **three** of the following antiderivatives. If you work on more than three you will get credit for the best three. Continue your work on the back of this page if you need more room. (45 points)

   a) $\displaystyle\int x^2 \ln x\,dx$ b) $\displaystyle\int \cos^3 2x \sin^2 2x\,dx$ c) $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\,dx$

   d) $\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$ e) $\displaystyle\int \frac{3x^2-x-4}{(x+2)(x^2+1)}\,dx$

$$|I-T_n| \le \frac{K_2(b-a)^3}{12n^2} \qquad |I-M_n| \le \frac{K_2(b-a)^3}{24n^2} \qquad |I-S_n| \le \frac{K_4(b-a)^5}{180n^4}$$

2. Write a short essay on the trapezoid error estimate formula. (15 points) Specifically, your essay should address at least the following:
   - The meanings of the notation I, T_n, K_2, and n,
   - What quantity is being approximated and what quantity is doing the approximation,
   - How the error is represented in the formula,
   - Why or how the formula implies that under certain circumstances the error is small and what those circumstances are, and
   - Why the corresponding error formula for Simpson's Rule leads us generally to expect that Simpson's rule will give a better approximation than the trapezoid rule.

3. Consider $\displaystyle\int_0^5 \frac{\sin x}{1+4^x}\,dx$. In this problem you may **not** use *Mathematica*'s built-in approximation of the integral, that is, you may not use the number that *Mathematica* gives when you add **//N** to the integral. You will need to use *Mathematica* for other purposes, and you should indicate what you use it for, how you use it, and the conclusions you draw from it.
   a) Determine the number of subintervals needed in order to guarantee that the midpoint sum approximation of the integral is good to three decimal places, that is, the error should be less than .0005. (12 points)
   b) Based on part a), what approximation do you get for the value of the integral? (3 points)

4. Consider the improper integrals $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx$ and $\displaystyle\int_1^\infty \frac{3x^2}{x^5+\sqrt{x}+1}\,dx$. Suppose you know that the first integral equals 1/2. Explain why this implies that the second integral converges, and give an upper bound for the value of the second integral. (5 points)

5. Consider $\displaystyle\int_0^2 \frac{1}{\sqrt{x}}\,dx$. Explain briefly why this integral is improper. Then express it as a limit and find its value. (10 points)

6. Evaluate the following limits. Briefly explain. (10 points)

   a) $\displaystyle\lim_{x\to\infty} \frac{\sin 2x}{x}$ b) $\displaystyle\lim_{x\to 0} \frac{\sin 2x}{x}$

Selected answers and hints.

1. See what *Mathematica* gets.
3. a) $K_2 = 0.8$, $n = 92$ b) 0.262835
4. The upper bound is $3/2$. See what the integrand of the second integral is less than. It's *not* less than the integrand of the first integral.
5. $2\sqrt{2}$ Expressing the integral as an appropriate limit is an essential part of the problem.
6. Be careful and don't say something that has no meaning.