Math 112 **Exam 3** **Name:**

2 December 2011 100 Points

"Show enough work to justify your answers."

READ THIS !! You may use *Mathematica* to help you think on any problem. You may use it as part of your solution only on those problems that indicate it. If you have trouble with *Mathematica*, be sure to ask. You may not use other software, the Internet, or other on-line resources.

READ THIS !! There are two parts. In Part I you are to do both problems. In Part II, you have a choice of problems.

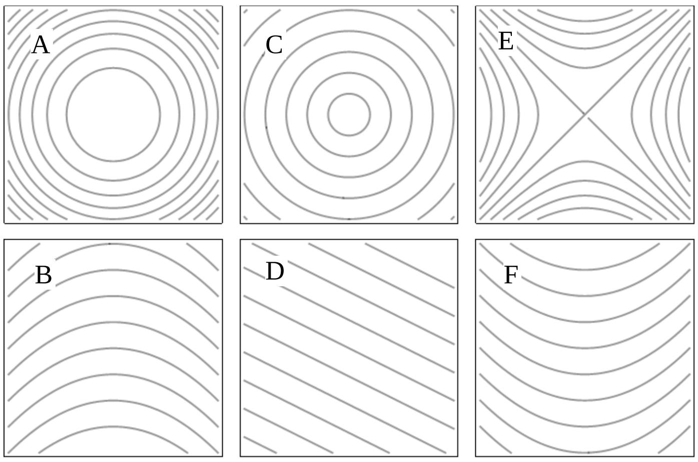
Part I. Do both problems. (25 points)

1. Multiple choice. For each function, determine which picture (if any) is a plot of some of its level curves. Put the letter of the picture in the blank next to the function, or put N if no picture is correct. (15 points)

   (a) $f(x, y) = x + 2y$ _______ d) $k(x, y) = x^2 - y^2$ _______

   (b) $g(x, y) = x^2 + 2y$ _______ e) $p(x, y) = x^2 + y^2$ _______

   (c) $h(x, y) = xy$ _______



2. Evaluate (10 points): $\displaystyle\int_0^2 \int_0^{x^2} (x + y)\, dy\, dx$

Part II. Read Carefully! Do any **five** of the remaining problems. If you work on more than five, you will get credit for the best five. They are worth 15 points each. I suggest quickly reading though them to see what the problems are. (75 points)

3. Let $z = f(x, y) = x^2 + xy - y^2$.

   (a) Compute dz. (3 points)

   (b) Find an equation of the plane tangent to the graph of f at the point where $(x, y) = (1, 2)$. For full credit, write the equation in a form that displays the point of tangency. (5 points)

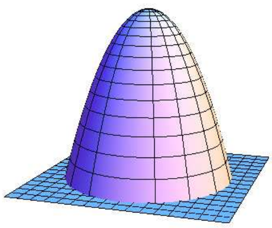
   (c) Find an equation of the level curve of f that passes through $(1, 2)$. (2 points)

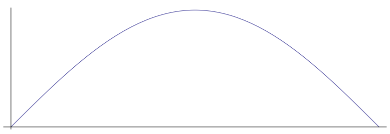
   (d) Find an equation of the line tangent to the level curve in the previous part at $(1, 2)$. For full credit, write the equation in a form that displays the point of tangency. (5 points)

4. Suppose that the variables u, v, and w are related by the equation $u^2 - v + w^3 = \sin(uv)$. Thinking of u as a function of v and w, find $\frac{\partial u}{\partial v}$ and $\frac{\partial u}{\partial w}$ in terms of u, v, and w.

5. Find the maximum and minimum values of the product of three non-negative numbers x, y, and z for which $x + 2y + 3z = 18$.

6. The picture (below left) shows the portion of the surface $z = 9 - x^2 - y^2$ that is above the xy-plane. Find the volume enclosed. There are two approaches to this. One is to do a single integral, noting that the horizontal cross sections are circles. The other is to do a double integral over the base of the solid in the xy-plane. If you use a double integral, you may use *Mathematica* to evaluate it. Be sure to write down the integral you evaluate and not just the result.





7. Write a ready-to-evaluate integral that gives the length of one arch of the graph of $y = \sin x$ (above right). Use *Mathematica* to evaluate it. What does *Mathematica* use to represent the exact value? What is *Mathematica*'s numerical approximation? (Remember to write $\sin x$ and $\cos x$ as Sin[x] and Cos[x] in *Mathematica*.)

8. Find both stationary (critical) points of $f(x, y) = x^2 - 4xy + \frac{1}{3}y^3 + 12y$. For each stationary point, determine if it is a local maximum, local minimum, or saddle point.

9. Sketch the region of integration and reverse the order of integration. Do not evaluate.

$$\int_0^2 \int_0^{x^2} e^x \sin y \, dy \, dx$$

Selected answers and hints.

1. Set each equal to a constant and determine the type of curve you get. Use this to eliminate possibilities.

2. 36/5

3. (b) $z + 1 = 4(x - 1) - 3(y - 2)$
   (c) $x^2 + xy - y^2 = -1$
   (d) This is like (b) except that z doesn't change when you move along the level curve, so $dz = 0$.

4. $\dfrac{\partial u}{\partial w} = \dfrac{3w^2}{v\cos(uv) - 2u}$

5. The maximum is 36. The minimum is 0.

6. $81\pi/2$

7. $2\sqrt{2}\,\mathrm{EllipticE}(1/2)$

8. One is a saddle point. One is a local minimum.

9. $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 e^x \sin y \, dx \, dy$