Math 112  Exam 3  Name:
2 December 2011  100 Points
“Show enough work to justify your answers.”

READ THIS !! You may use Mathematica to help you think on any problem. You may use it as part of your solution only on those problems that indicate it. If you have trouble with Mathematica, be sure to ask. You may not use other software, the Internet, or other on-line resources.

READ THIS !! There are two parts. In Part I you are to do both problems. In Part II, you have a choice of problems.

Part I. Do both problems. (25 points)

1. Multiple choice. For each function, determine which picture (if any) is a plot of some of its level curves. Put the letter of the picture in the blank next to the function, or put N if no picture is correct. (15 points)

   (a) \( f(x, y) = x + 2y \) ________

   (b) \( g(x, y) = x^2 + 2y \) ________

   (c) \( h(x, y) = xy \) ________

   (d) \( k(x, y) = x^2 - y^2 \) ________

   (e) \( p(x, y) = x^2 + y^2 \) ________

![Pictures A, B, C, D, E, F]

2. Evaluate (10 points):
\[
\int_0^2 \int_0^{x^2} (x + y) \, dy \, dx
\]
**Part II. Read Carefully!** Do any five of the remaining problems. If you work on more than five, you will get credit for the best five. They are worth 15 points each. I suggest quickly reading though them to see what the problems are. (75 points)

3. Let \( z = f(x, y) = x^2 + xy - y^2 \).
   
   (a) Compute \( dz \). (3 points)
   
   (b) Find an equation of the plane tangent to the graph of \( f \) at the point where \( (x, y) = (1, 2) \). For full credit, write the equation in a form that displays the point of tangency. (5 points)
   
   (c) Find an equation of the level curve of \( f \) that passes through \( (1, 2) \). (2 points)
   
   (d) Find an equation of the line tangent to the level curve in the previous part at \( (1, 2) \). For full credit, write the equation in a form that displays the point of tangency. (5 points)

4. Suppose that the variables \( u, v, \) and \( w \) are related by the equation \( u^2 - v + w^3 = \sin(uv) \). Thinking of \( u \) as a function of \( v \) and \( w \), find \( \frac{\partial u}{\partial v} \) and \( \frac{\partial u}{\partial w} \) in terms of \( u, v, \) and \( w \).

5. Find the maximum and minimum values of the product of three non-negative numbers \( x, y, \) and \( z \) for which \( x + 2y + 3z = 18 \).

6. The picture (below left) shows the portion of the surface \( z = 9 - x^2 - y^2 \) that is above the \( xy \)-plane. Find the volume enclosed. There are two approaches to this. One is to do a single integral, noting that the horizontal cross sections are circles. The other is to do a double integral over the base of the solid in the \( xy \)-plane. If you use a double integral, you may use Mathematica to evaluate it. Be sure to write down the integral you evaluate and not just the result.

7. Write a ready-to-evaluate integral that gives the length of one arch of the graph of \( y = \sin x \) (above right). Use Mathematica to evaluate it. What does Mathematica use to represent the exact value? What is Mathematica’s numerical approximation? (Remember to write \( \sin x \) and \( \cos x \) as \( \text{Sin}[x] \) and \( \text{Cos}[x] \) in Mathematica.)

8. Find both stationary (critical) points of \( f(x, y) = x^2 - 4xy + \frac{1}{3}y^3 + 12y \). For each stationary point, determine if it is a local maximum, local minimum, or saddle point.

9. Sketch the region of integration and reverse the order of integration. Do not evaluate.
\[
\int_0^2 \int_0^{x^2} e^x \sin y \, dy \, dx
\]
Selected answers and hints.

1. Set each equal to a constant and determine the type of curve you get. Use this to eliminate possibilities.

2. $36/5$

3. (b) $z + 1 = 4(x - 1) - 3(y - 2)$
   (c) $x^2 + xy - y^2 = -1$
   (d) This is like (b) except that $z$ doesn’t change when you move along the level curve, so $dz = 0$.

4. $\frac{\partial u}{\partial w} = \frac{3w^2}{v \cos(uv) - 2u}$

5. The maximum is 36. The minimum is 0.

6. $81\pi/2$

7. $2\sqrt{2} \text{EllipticE}(1/2)$

8. One is a saddle point. One is a local minimum.

9. $\int_0^4 \int_0^{2\sqrt{y}} e^x \sin y \, dx \, dy$