Math 112 **Exam 2** **Name:**

10 October 2011 100 Points

No *Mathematica.*

"Show enough work to justify your answers."

Read carefully: Do any **five** problems. If you work on more than five, you will get credit for the best five. Read them all quickly to see what they are like. (20 points each)

Note: Every time you claim a series converges or diverges, to get full credit, you must give full reasoning. If that reasoning involves a test, you must state the name of the test and show the details.

Instructions for 1–5. Determine if the following converges absolutely, converges conditionally, or diverges. You may give an educated guess for 5 points partial credit.

1. $\displaystyle\sum_{n=5}^{\infty}(-1)^n\frac{1}{2^n-n^2}$

2. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{10^n+5}{n!}$

3. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{2^n-n}$

4. $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\ln n}$

5. $\displaystyle\sum_{n=1}^{\infty}\tan^n(1/n)$

6. A ball is launched upwards from the ground to a height of 100 feet. Every time it bounces it reaches a height 3/5 times the previous height. What is the total distance the ball travels?

7. The improper integral $\displaystyle\int_1^{\infty}\frac{x}{3x^4+10}\,dx$ can be shown to converge by comparing it with $\displaystyle\frac{1}{3}\int_1^{\infty}\frac{1}{x^3}\,dx$. (You do not need to do this.) Use this information to find a value of b such that $\displaystyle\int_b^{\infty}\frac{x}{3x^4+10}\,dx$ is less than $1/1000$. Give full justification.

8. The series $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^3+5}$ can be shown to converge by comparing it with $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$. (You do not need to do this.) Suppose $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^3+5}$ is approximated by $\displaystyle\sum_{n=1}^{100}\frac{n}{n^3+5}$. Determine an upper bound for the error made by this approximation. Give full justification.

Selected answers and hints.

1.–5. Of the series in these five problems, three of them converge absolutely, one converges conditionally, and one diverges.

6. 500 feet

7. $b = 10\sqrt{5/3} \approx 12.91$ works.

8. The error is less than $1/100$.