Math 112 **Final Exam** **Name:**

16 December 2009 200 Points

No *Mathematica.*

"Show enough work to justify your answers."

1. Do **all six** parts of this problem, showing all work on these pages. (10 points each)
   (a) Evaluate: $\int x \sin 3x \, dx$
   (b) Evaluate: $\int_0^1 \int_0^x (x+y) \, dy \, dx$
   (c) Evaluate: $\int (x+3)\sqrt{x^2+6x-7} \, dx$
   (d) Give the power series expansion about 0 of the function $f(x) = \dfrac{\sin(x^2)}{x}$. Give at least six non-zero terms.
   (e) Consider the point P with Cartesian coordinates $(x, y) = (\sqrt{3}, 1)$. Give two sets of polar coordinates for P, one with $r > 0$ and one with $r < 0$.
   (f) Let $f(x, y, z) = z \sin(x/y)$. Compute f_{xzx}.

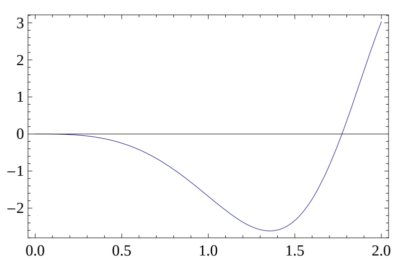
READ THIS !! Do any **seven** of the remaining problems. If you work on more than seven, you will get credit for the best seven. (20 points each) Please work each problem on a separate sheet of paper. Put the problem number in the upper right corner. Avoid writing in the upper left corner where the staple will go.

$$|I - T_n| \le \frac{K_2(b-a)^3}{12n^2} \qquad |I - M_n| \le \frac{K_2(b-a)^3}{24n^2} \qquad |I - S_n| \le \frac{K_4(b-a)^5}{180n^4}$$

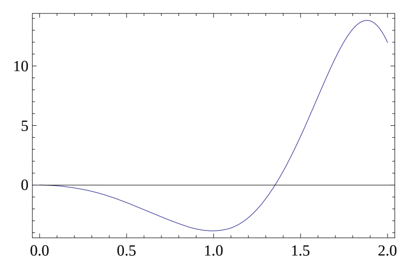
2. Clearly explain in words and pictures how the concavity of f is related to a trapezoid sum T_n being an overestimate or underestimate of $\int_a^b f(x)\,dx$.

3. Suppose you want to approximate the integral $\displaystyle\int_0^2 \cos(x^2)\,dx$ with an error less than 1/1000.
   (a) Determine with proof how many subintervals are needed to do this for a midpoint sum.
   (b) Determine with proof how many subintervals are needed to do this for a Simpson's Rule sum.

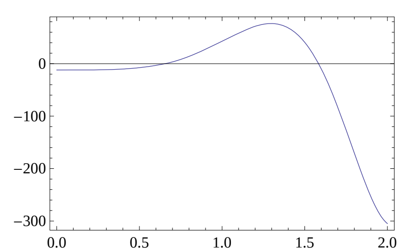
   The graphs of f', f'', and $f^{(4)}$ are shown, where $f(x) = \cos(x^2)$.



Graph of f'



Graph of f''



Graph of $f^{(4)}$

4. Consider the improper integral $\displaystyle\int_b^\infty \frac{x^2}{x^5+x^3+10}\,dx$.

   (a) Explain why $\displaystyle\frac{x^2}{x^5+x^3+10} < \frac{1}{x^3}$ for $x > 0$. (5 points)

   (b) Find a value for b with proof such that $\displaystyle\int_b^\infty \frac{x^2}{x^5+x^3+10}\,dx < \frac{1}{100}$. (15 points)

5. A ball is dropped from a height of 8 feet. Each time it bounces it comes up to 3/4 the height of the previous bounce. Use series to determine how far the ball travels. It may be helpful to use separate series for the up and down directions.

6. Evaluate: $\displaystyle\lim_{x\to 0^+}(1+2x)^{1/x}$

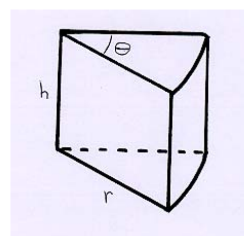
7. Determine, with proof, if $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{n^2}{n^3-1}$ converges absolutely, converges conditionally, or diverges. To get full credit you must indicate which tests you use, and give full details of their use. It may be helpful to consider the function $f(x) = x^2/(x^3-1)$.

8. Let $z = f(x,y) = x^2/y$.

   (a) Compute dz.

   (b) Find the equation of the plane tangent to the graph of f at the point where $(x,y) = (2,-1)$. Leave your answer in a form that exhibits the point of tangency.

   (c) Find the equation of the line tangent to the level curve of f that passes through $(2,-1)$. Leave your answer in a form that exhibits the point of tangency.



9. A manufacturer is packaging cheese wedges in the shape pictured. If the volume of the wedge is 27 cubic inches, find the values of r, θ, and h that give the least surface area. (The area of the base is $\frac{1}{2}r^2\theta$, and the area of the curved part of the surface is $r\theta h$.)

10. Consider the double integral $\displaystyle\int_0^2\int_{x^2}^4 \sin(e^x+\tan y)\,dy\,dx$.

    (a) Make a clear sketch of the region of integration—not too small. Shade the region.

    (b) Reverse the order of integration. Do not evaluate.

11. Let R be the region between the circles $x^2+y^2=1$ and $x^2+y^2=9$ and to the right of the y-axis. Evaluate the following integral.
    $$\iint_R \frac{1}{x^2+y^2}\,dA$$

12. Find the area bounded by the graph of $f(x) = x^2-4x+3$ and the x-axis between $x=0$ and $x=4$. (Think before you compute!)

13. Consider the ellipse $4x^2+9y^2=36$. Write a simplified, ready-to-evaluate integral that gives the circumference of the ellipse. Do not evaluate.

Have a good break!

Selected hints and answers.

1. (a) – (c) and (f) See what *Mathematica* gets.
   (d) $x - x^5/3! + x^9/5! - x^{13}/7! + \dots$
   (e) $(2, \pi/6)$, $(-2, 7\pi/6)$ (There are other correct answers.)

3. (a) $n \geq 71$ (b) $n \geq 16$

4. (b) $b \geq \sqrt{50}$

5. 56 feet

6. e^2

7. Remember that conditional convergence requires that you prove two things.

8. The tangent plane to the graph and the tangent line to the level curve are answers to two questions that are nearly the same. One step along the way to getting the tangent plane is $dz = -4\,dx - 4\,dy$. When you move a tiny distance on the graph, dz does not have to be zero, but when you move a tiny distance along the level curve, dz does have to be zero.
   Tangent plane: $z + 4 = -4(x - 2) - 4(y + 1)$.
   Tangent line: $0 = -4(x - 2) - 4(y + 1)$ or $0 = (x - 2) + (y + 1)$.

9. $r = h = 3$, $\theta = 2\,\text{rad}$. You can use Lagrange multipliers, but it's easier if you eliminate a variable.

10. $\int_0^4 \int_0^{\sqrt{y}} \sin(e^x + \tan y)\,dx\,dy$

11. $\pi \ln 3$

12. 4

13. $\dfrac{2}{3} \displaystyle\int_{-3}^{3} \frac{\sqrt{81 - 5x^2}}{\sqrt{9 - x^2}}\,dx$ or $\int_0^{2\pi} \sqrt{9\sin^2 t + 4\cos^2 t}\,dt$