Math 112 **Exam 3** **Name:**

4 December 2009 100 Points

No *Mathematica*

"Show enough work to justify your answers."

READ THIS !! Do the first problem. You then get a choice of the remaining problems.

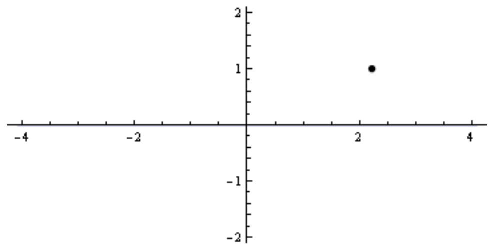
1. Evaluate the following, showing all steps. (10 points)

$$\int_{-1}^{0}\int_{0}^{x^2} y\left(5+\frac{8}{x}\right)\,dy\,dx$$

READ THIS !! Do any **six** of the remaining problems. If you work on more than six, you will get credit for the best six. They are worth 15 points each.

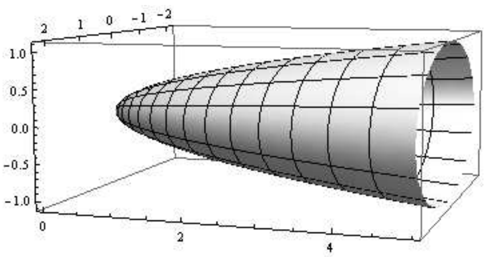
2. Sketch the region of integration for the integral in Problem 1, and rewrite the integral with the order of integration reversed. Draw a clear picture, not too small.

3. Let $z = f(x,y) = x^2 + 4y^2$. Sketch three level curves of f: for $z = 4$, $z = 16$, and the level curve passing through $(\sqrt{5}, 1)$. Sketch them all on the same coordinate axes. Label each curve with its z-value.



4. The graph of $y = x^2 + 4z^2$ is shown.

   (a) Draw the curve on the graph obtained by slicing the graph with the plane $y = 4$. Determine the equation of the curve and whether the curve is a circle, ellipse, parabola, or hyperbola.



   (b) Draw the curve on the graph obtained by slicing the graph with the plane $x = 1$. Determine the equation of the curve and whether the curve is a circle, ellipse, parabola, or hyperbola.

   (c) Determine the coordinates of the two points where the curves intersect.

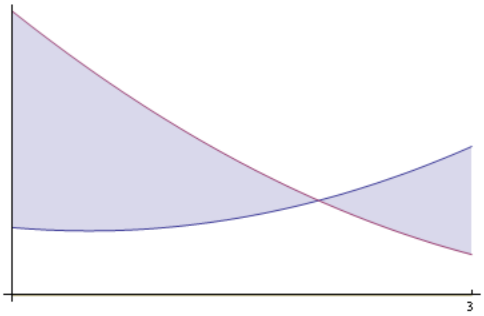
5. Let $z = f(x,y) = x^2/y$.

   (a) Compute dz.

   (b) Find the equation of the plane tangent to the graph of f at the point where $(x,y) = (2,-1)$. Leave your answer in a form that exhibits the point of tangency.

   (c) Find the equation of the line tangent to the level curve of f that passes through $(2,-1)$. Leave your answer in a form that exhibits the point of tangency.

6. The following picture shows the functions $f(x) = x^2 - x + 5$ and $g(x) = x^2 - 9x + 21$ graphed on the interval $0 \le x \le 3$. Find the amount of shaded area.



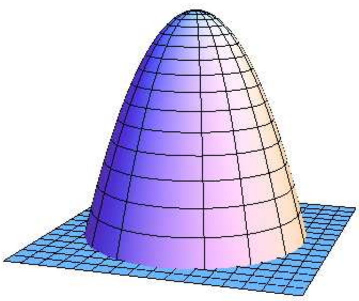
7. Find all critical points of the following function in the first and second quadrants, and determine if they are local minima, local maxima, or saddle points.

   $$f(x,y) = \frac{1}{x} - \frac{1}{y} + 4x - y$$

8. Find the maximum and minimum values of $f(x,y) = xy$, and their locations, on the elliptical disk $x^2 + 2y^2 \le 4$.

9. The equation $z^3 = x^2 - yz^2$ defines some surface in three-dimensional space. Verify that the point $(1,2,-1)$ is on the surface, and approximate the z-coordinate of the nearby point $(1.1, 1.8, z)$ on the surface.

10. The picture below shows the portion of the surface $z = 16 - x^2 - y^2$ that is above the xy-plane. Find the volume enclosed. There are two approaches to this. One is to do a single integral, noting that the horizontal cross sections are circles. The other is to do a double integral over the base of the solid in the xy-plane. The double integral is easier to do if you convert it to polar coordinates.



Selected answers and hints.

1. $-1/2$
2. $\int_0^1 \int_{-1}^{-\sqrt{y}} y \left(5 + \frac{8}{x}\right)\, dx\, dy$
3. The level curve passing through $(\sqrt{5}, 1)$ is $x^2 + 4y^2 = 9$.
4. The view is the standard one, with the y-axis pointing to the right. The curves are an ellipse and a parabola in parts (a) and (b), respectively.
5. Tangent plane: $z + 4 = -4(x - 2) - 4(y + 1)$. Tangent line: $0 = -4(x - 2) - 4(y + 1)$.
6. 20
7. The two critical points are $(\pm 1/2, 1)$. One is a local maximum, one is a saddle point.
8. There is one interior critical point and four boundary critical points. The function takes a maximum value of $\sqrt{2}$ at $(\sqrt{2}, 1)$, and $(-\sqrt{2}, -1)$. It takes a minimum value of $-\sqrt{2}$ at $(-\sqrt{2}, 1)$, and $(\sqrt{2}, -1)$.
9. $z \approx -1.4$
10. 128π