Math 112 **Exam 2** **Name:**

12 October 2009 100 Points

No *Mathematica*.

"Show enough work to justify your answers."

Read carefully: Do any **five** problems. If you work on more than five, you will get credit for the best five. Read them all quickly to see what they are like. (20 points each)

Note: Every time you claim a series converges or diverges, to get full credit, you must give full reasoning. If that reasoning involves a test, you must state the name of the test and show the details.

1. Determine if the following converges absolutely, converges conditionally, or diverges. You may give an educated guess for 5 points partial credit.
$$\sum_{n=0}^{\infty}(-1)^n\frac{100^n}{n!}$$

2. Determine if the following converges absolutely, converges conditionally, or diverges. You may give an educated guess for 5 points partial credit.
$$\sum_{n=0}^{\infty}(-1)^n\frac{n+3}{2n-5}$$

3. Determine if the following converges absolutely, converges conditionally, or diverges. You may give an educated guess for 5 points partial credit.
$$\sum_{n=1}^{\infty}\frac{n}{n^2+5}$$

4. Determine if the following converges absolutely, converges conditionally, or diverges. You may give an educated guess for 5 points partial credit.
$$\sum_{n=5}^{\infty}(-1)^n\frac{1}{\sqrt{n}-2}$$

5. Explain why the following series converges and find its sum. Note: There is only one type of series you know the sum of, so this must be related somehow.
$$\sum_{n=0}^{\infty}\frac{1+2^n}{3^n}$$

6. Find the full interval of convergence of the following power series.
$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2}$

7. The improper integral $\int_1^{\infty} \frac{1}{\sqrt{x^4+5x^3+7}}\,dx$ can be shown to converge by comparing it with $\int_1^{\infty} \frac{1}{x^2}$. (You do not need to do this.) Use this information to find a value of b such that $\int_b^{\infty} \frac{1}{\sqrt{x^4+5x^3+7}}\,dx$ is less than $1/1000$. Give full justification.

8. Suppose $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is approximated by $\sum_{n=1}^{15} \frac{1}{n^3}$. Determine an upper bound for the error made by this approximation. Give full justification.

9. The purpose of this problem is to motivate the surprising sum of the alternating harmonic series.

   (a) Explain why the following formula is true.

   $$1 - x + x^2 - x^3 + x^4 - x^5 + \ldots = \frac{1}{1+x} \qquad \text{for } |x| < 1$$

   (b) Find an antiderivative **separately** for each side of the equation in part (a).

   Left side:

   Right side:

   (c) Your two antiderivatives might differ by a constant. Determine a constant you can add to one of them so that they are equal and write an equation. (Consider what happens when $x = 0$.)

   (d) Set $x = 1$ to find the sum of the alternating harmonic series!

Selected answers and hints.

1.–4. Of the series in these four problems, two of them converge, one absolutely and one conditionally, and two diverge.

5. The sum is $9/2$. Hint: There is only one type of series you know the sum of. (Is there an echo on this sample exam?) Think about what you can do to express this series in terms of that type of series.

6. $2 \leq x \leq 4$. This series converges absolutely at both endpoints (this was not part of the question).

7. $b = 1000$ works.

8. The error is less than $1/450 = .00\overline{22}$.

9. (c) $x - x^2/2 + x^3/3 - x^4/4 + x^5/5 - x^6/6 + \ldots = \ln(1 + x)$

   (d) $1 - 1/2 + 1/3 - 1/4 + 1/5 - 1/6 + \ldots = \ln 2$