Math 112 **Exam 1** **Name:**

21 September 2009 100 Points

"Show enough work to justify your answers."

Read carefully: This exam has two parts. You are to do **all** of the problems on Part I. On Part II you will have a choice of problems. You may use *Mathematica* or a calculator in any problem to help you think. You may use them as part of your reasoning only in the problems that indicate so.

Part I. Do all of the problems on this page. (25 points total)

1. Evaluate (9 points): $\int \sin x \sqrt{5 \cos x + 3}\, dx$

2. Evaluate (8 points): $\lim_{x \to 2} \dfrac{8 - x^3}{x^2 - x - 2}$

3. The following table gives values of a function f. Use this to compute the left-endpoint approximation of $\int_0^2 f(x)\, dx$ with four subintervals. Show enough work so I can tell what you are thinking. Use *Mathematica* or a calculator to do the arithmetic. (8 points)

x	0.0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0
$f(x)$	1.0	1.29	1.56	1.78	1.93	1.99	1.97	1.86	1.67

Part II. Do **any five** of the remaining problems. If you work on more than five, you will get credit for the best five. Suggestion: Read them all quickly to see what they are like. (15 points each; 75 points total)

4. Evaluate $\int x^2 e^{3x}\, dx$

5. Evaluate $\int \dfrac{1}{\sqrt{x^2 + 16}}\, dx$

6. Evaluate: $\int \cos^5(3x)\, dx$

$$|I - M_n| \le \frac{K_2(b-a)^3}{24n^2} \qquad\qquad |I - S_n| \le \frac{K_4(b-a)^5}{180n^4}$$

7. Consider approximating $\int_0^3 1.1^{-x^2}\,dx$ with the midpoint rule and Simpson's rule. You will need *Mathematica* in all parts of this problem. If you need help, ask. Express your answers to as many digits as *Mathematica* gives. To get full credit in parts (b) and (c), you must be completely clear how you make all of your conclusions. In particular, you will need to draw some graphs and indicate what they are.

   (a) Compute the values of the midpoint rule and Simpson's rule approximations of the integral using 12 subintervals. (3 points)

   Midpoint approx = Simpson's approx =

   (b) What does the midpoint rule error estimate guarantee about the error made by the midpoint rule? (6 points)

   (c) What does the Simpson's rule error estimate guarantee about the error made by Simpson's rule? (6 points)

8. Consider the midpoint rule error estimate.

   (a) What does I represent? (2 points) Answer here:

   (b) What does n represent? (1 points)

   (c) What does M_n represent? (2 points)

   (d) What does K_2 represent? (3 points)

   (e) What part of the formula represents the error? (3 points)

   (f) Comparing the two error estimate formulas above, what features of the formulas imply that Simpson's rule generally gives a better approximation than the midpoint rule? Briefly explain. (4 points)

9. Evaluate $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx$. To do this, let $u = 1 + \sqrt{x}$. Before computing how du and dx are related, solve the equation for x. Then compute dx in terms of u and du. This will allow you to complete the substitution.

10. Evaluate: $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{x} - \frac{2}{\sin x}\right)$

11. Consider $\displaystyle\int_2^5 \frac{1}{(x-2)^2}\,dx$.

    (a) Briefly explain why this is an improper integral. (5 points)

    (b) Evaluate it and determine if it converges or diverges. For full credit, you must express it as a limit. (10 points)

Selected answers and hints

1. See what *Mathematica* gets.

2. -4

3. 3.23

4.–6. See what *Mathematica* gets.

7. This problem requires the use of the *Mathematica* notebook NumericalInt.nb in the folder N:/Math/Math112/Foote.

   (a) Simpson's: 2.32445

   (b) The error for the Midpoint Rule approximation is less than .0015625. A complete answer for this part includes a sketch of the graph of f'' and a clear indication of the value of K_2 used.

9. See what *Mathematica* gets.

10. $-\infty$

11. (b) See what *Mathematica* gets.