Math 112 **Exam 3** **Name:**

29 November 2006 100 Points

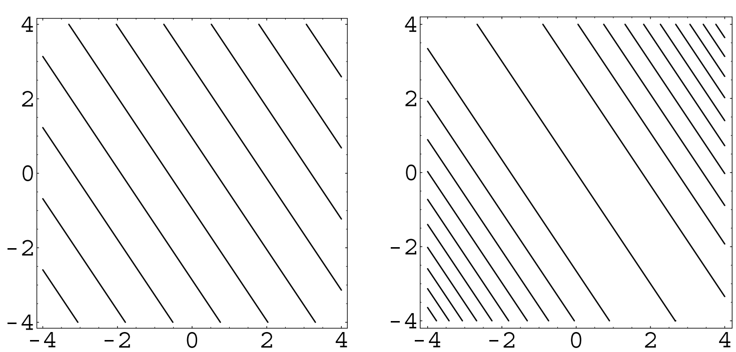
"Show enough work to justify your answers."

READ THIS !!! You are to do problems 1 and 2, and then any four of the remaining problems.

You may use *Mathematica* on any problem to help you think, however, you may **not** use it as as part of a solution except as noted. If you have trouble with *Mathematica*, please ask.

1. Evaluate the following, showing all steps. Do your work on this page. (10 points)
$$\int_0^3 \int_0^{\sqrt{x}} y \sin(\pi x^2)\, dy\, dx$$

2. The level curves for the functions $f(x, y) = 3x + 2y$ and $g(x, y) = (3x + 2y)^2$ are shown. Indicate which function goes with which set of level curves. Explain how you know. Do your work on this page. (10 points)



READ THIS !!! Do any **four** of the remaining problems. If you work on more than four, you will get credit for the best four. (20 points each)

Please begin each problem on a new sheet of paper, except as noted. Avoid writing in the upper left corner of the page (where the staple will go). When you are done, put the pages in order with the problem number in the *upper right* corner, and staple them to your exam.

3. Sketch the region of integration and reverse the order of integration for the integral in problem 1.

4. Suppose that $w = \dfrac{xy^2}{z}$.

   (a) Compute dw in terms of x, y, z, and their differentials.

   (b) Show that $\dfrac{dw}{w} = \dfrac{dx}{x} + 2\dfrac{dy}{y} - \dfrac{dz}{z}$. Do this either by manipulating your answer to part (a) or by another method.

5. Find all critical points of the following function in the first and second quadrants, and determine if they are local minima, local maxima, or saddle points.

   $f(x, y) = \dfrac{1}{x} - \dfrac{1}{y} + 4x - y$

6. A rectangular box has a volume of 20 cubic inches. The material for the sides, bottom, and top cost \$1, \$2, and \$3 per square inch, respectively. Find the dimensions and cost of the least expensive box. Do all calculus by hand. You may use *Mathematica* to do the algebra. If you use *Mathematica*, clearly indicate what you use it for and what your conclusions are.

7. The following table gives values of a function f for $0 \le x \le 1$, $.3 \le y \le .7$. Use these values to give an approximation of $\int_{0.1}^{0.5} \int_{0.4}^{0.6} f(x, y)\, dy\, dx$. Use $\Delta x = .2$ and $\Delta y = .1$. Show enough detail so I can tell what you are doing. In particular, draw the region of integration in the xy-plane (separate from the table), and clearly indicate the subrectangles you are using, their dimensions, and the value of the function you are using for each subrectangle. You may use *Mathematica* or a calculator to do the arithmetic.

0.7	-0.3	0.1	0.5	0.9	1.3	1.7
0.6	-0.4	0.	0.4	0.8	1.2	1.6
0.5	-0.5	-0.1	0.3	0.7	1.1	1.5
0.4	-0.6	-0.2	0.2	0.6	1.	1.4
0.3	-0.7	-0.3	0.1	0.5	0.9	1.3
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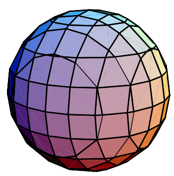
	0	0.2	0.4	0.6	0.8	1.

8. For each equation, determine which picture is its graph, and briefly explain how you know. **Do your work on this page.**

(a) $z = x^2 + y^2$ ______ c) $z^2 - 1 = x^2 + y^2$ ______

(b) $z = y^2 - x^2$ ______ d) $x^2 + y^2 + z^2 = 4$ ______

A



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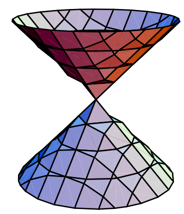
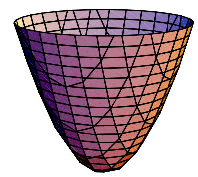
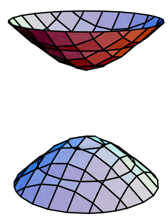


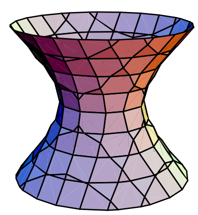
C



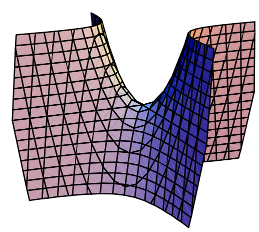
D



E



F



Selected answers and hints.

1. See what *Mathematica* gets.

2. Note that f is linear. Think about the graph of a linear function and the consequence for its level curves.

3. $\int_0^{\sqrt{3}} \int_{y^2}^{3} y \sin(\pi x^2)\, dx\, dy$

5. There are two critical points. One is a saddle point; one is a local maximum.

6. Note that the length, width, and height of the box are not independent. Therefore you either need to eliminate a variable or use Lagrange multipliers. The cost of the least expensive box is \$60.

7. The most frequent error is to have too many subrectangles. With $\Delta x = .2$ and $\Delta y = .1$, how many subrectangles should there be? The value of the approximation for the integral depends on what function values are used. If you use the average of the function values at the midpoints of the top and bottom of each subrectangle, the approximate Riemann sum works out to .008.

8. Rule out alternatives by considering the types of traces you get by setting some variable to a constant.