Math 112 **Exam 2** **Name:**

11 October 2006 100 Points

"Show enough work to justify your answers."

READ THIS !! Do any **five** problems. If you work on more than five, you will get credit for the best five. They are worth 20 points each. I suggest quickly reading though the exam to see what the problems are.

You may use *Mathematica* on any problem to help you think, however, you may **not** use it as as part of a solution except as noted. If you have trouble with *Mathematica*, please ask.

Please begin each problem on a new sheet of paper. Avoid writing in the upper left corner of the page (where the staple will go). When you are done, put the pages in order with the problem number in the *upper right* corner, and staple them to this page.

1. Explain why $\int_0^1 \ln x\,dx$ is improper and evaluate it by hand showing all steps. If you don't remember how to do the antiderivative $\int \ln x\,dx$, you may use *Mathematica* to evaluate it for partial credit.

2. Find a value b such that $\displaystyle\int_b^\infty \frac{x}{x^3+7x^2+5x+3}\,dx$ is less than $1/100$.

3. Explain in your own words the difference between an infinite sequence and an infinite series. Explain what convergence means for each. You may refer to specific examples if you like.

4. The series $\displaystyle\sum_{k=0}^{\infty} \frac{3^k}{2^k+5^k}$ converges (you do not need to prove this). Suppose this series is approximated by the finite sum $\displaystyle\sum_{k=0}^{10} \frac{3^k}{2^k+5^k}$. Give a reasonable upper bound on the error made by this approximation.

5. Consider the series $\sum_{k=0}^{\infty} \frac{(-2)^k}{k!}$.

   (a) State whether this series converges absolutely, converges conditionally, or diverges. No work required; no partial credit. (5 points)

   (b) Carefully justify your claim in part (a). (15 points)

6. Consider the series $\sum_{k=1}^{\infty} \frac{k}{k^3 - 10}$.

   (a) State whether this series converges absolutely, converges conditionally, or diverges. No work required; no partial credit. (5 points)

   (b) Carefully justify your claim in part (a). (15 points)

7. Consider the series $\sum_{k=10}^{\infty} \frac{(-1)^k}{\sqrt{k} - 3}$.

   (a) State whether this series converges absolutely, converges conditionally, or diverges. No work required; no partial credit. (5 points)

   (b) Carefully justify your claim in part (a). (15 points)

8. Determine the full interval of convergence for the following power series, including endpoints: $\sum_{k=1}^{\infty} \frac{(x-3)^k}{k^2\, 2^k}$

9. Suppose that $\sum_{k=0}^{\infty} a_k (x-3)^k$ converges when $x = 5$ and diverges when $x = -5$. For each of the following, say if the statement *must* be true, *might* be true, or *cannot* be true. Briefly explain. (5 points each)

   (a) The series converges when $x = 2$.

   (b) The series converges when $x = 0$.

   (c) The series converges when $x = 11$.

   (d) The series converges when $x = 12$.

Selected answers and hints.

1. After evaluating the antiderivative, you get an indeterminant form at $x = 0$. This must be expressed as a limit to be evaluated.

2. $b \geq 100$

4. The error is less than 0.0091.

5. You don't need the AST.

6. To get the desired comparison, you need to compare it to $\sum 2/k^2$. You need to determine the values of k for which the desired inequality is valid. The ratio test won't work. It is always inconclusive on a series similar to a p-series.

7. To prove conditional convergence you need to prove *two* things. See the handout on using convergence tests.

8. For this series, convergence at one of the endpoints implies absolute convergence at the other endpoint, so you don't need the AST.

9. To do this problem, you need to think about what the interval of convergence might be: it is at least the interval $1 < x \leq 5$ and at most the interval $-5 < x \leq 11$.