**Math 112** **Exam 1** **Name:**

18 September 2006 100 Points

*"Show enough work to justify your answers."*

1. Evaluate **any three** of the following antiderivatives. If you work on more than three, you will get credit for the best three. Continue on the back of this page if you need more space. Do all work by hand, showing all steps. (12 points each)

   a) $\int \cos^3 x \, dx$ b) $\int x \sin(x+1) \, dx$

   c) $\int \sec^2(5x) \tan^2(5x) \, dx$ d) $\int \frac{1}{(x^2+9)^{3/2}} \, dx$

   e) $\int \frac{1}{1+\sqrt{x}} \, dx$ (Let $u = 1 + \sqrt{x}$. Solve this for $x$ before computing differentials.)

$$|I - L_n| \leq \frac{K_1(b-a)^2}{2n} \qquad |I - T_n| \leq \frac{K_2(b-a)^3}{12n^2} \qquad |I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}$$

2. Give short answers. (15 points) In the trapezoid sum error estimate formula,

   (a) What do $I$, $n$, and $T_n$ represent? (4 points)

   (b) Which part of the formula represents the exact value of the error? (3 points)

   (c) How does the formula imply that in a certain situation the error is small, and what is that situation? (4 points)

   (d) Why does the corresponding error formula for the left sum generally lead us to expect that the left sum will give a worse approximation than the trapezoid sum? (4 points)

3. You need to use *Mathematica* in this problem. To get full credit, you **must** clearly indicate what you use it for, how you use it, and the conclusions you draw from it. If you have trouble with *Mathematica*, ask for help. (14 points)

   Consider $\int_1^3 \frac{\cos(x^2)}{e^x} \, dx$.

   (a) Determine the number of subintervals needed for the trapezoid sum error estimate to guarantee that the trapezoid sum approximation of the integral is good to two decimal places, that is, the error should be less than .005. Clearly indicate how you arrive at your value for $K_2$. (10 points)

   (b) What is the approximate value of the integral based on part (a)? Give all decimal places so I can more easily tell what you have done. (4 points)

4. Let $I = \int_{-1}^{2} f(x)\,dx$, where $f$ has the values in the table. (15 points)

| $x$ | $-1.00$ | $-0.25$ | $0.50$ | $1.25$ | $2.00$ |
|---|---|---|---|---|---|
| $f(x)$ | $0.00$ | $2.65$ | $4.87$ | $6.83$ | $8.67$ |

   (a) Compute $T_4$. Give enough details so I can tell how you get your result. (8 points)

   (b) A plot of the data shows that the function is probably increasing and concave down. Rank $L_4$, $R_4$, $T_4$, $M_4$, and $I$ in increasing order. Briefly explain. (7 points)

5. Evaluate the following limits. If you use L'Hôpital's Rule, indicate why it is okay to use it. (10 points each)

   (a) $\displaystyle\lim_{x\to 0^+} \frac{x}{1-\cos x}$

   (b) $\displaystyle\lim_{x\to 0}(1+x)^{1/x}$

Selected answers and hints.

1. See what *Mathematica* gets.

3. (a) The smallest practical value for $K_2$ is 2.2 (2.1 is not big enough). The corresponding smallest value of $n$ is 18. (b) $T_{18} = -0.0496534$

4. (a) 14.01 (b) $L_4 < T_4 < I < M_4 < R_4$

   $L_4 < M_4 < R_4$ because $f$ is increasing.

   $T_4$ is between $L_4$ and $R_4$ becausse $T_4$ is the average of $L_4$ and $R_4$.

   $T_4 < I < M_4$ because $f$ is concave down.

5. Be careful not to use L'Hôpital's Rule when it's not valid!

   (a) $\infty$ (b) $e$