Math 111 **Final Exam** **Name:**

17 December 2010 200 Points

No *Mathematica*.

"Show enough work to justify your answers."

READ CAREFULLY! This exam has two parts. In Part I you are to work all of the problems. In Part II you are to work at least eight of the problems.

Part I. Do **all eight** of the problems in this part. (10 points each)

1. Evaluate: $\lim_{x\to\infty} \dfrac{x^3+5x-7}{x^2+\sqrt{x}-3x^3}$
2. Find dy/dx: $\sin x - x^2y = e^{2y}$
3. Compute the derivative and simplify: $f(x) = x^2\cos(x^3) + \dfrac{1}{\sqrt{x}} + \ln 2$
4. Find the equation of the line tangent to the graph of $f(x) = 50/x$ at $x = 5$.
5. Give the exact values of the following.

   $\sin(2\pi/3)$ $\cos(5\pi/4)$ $\tan(-\pi/6)$
6. Evaluate: $\displaystyle\int_{-1}^{3} (x^2 - x + 2)\,dx$
7. Find the formula for $f(x)$ where $f'(x) = 6x^2 - x + 5$ and $f(-2) = 0$.
8. Use Newton's Method with the function $f(x) = x^2 - x$ and the initial value of $x_1 = 2$. What is x_2? Express your answer as a decimal or a fraction. Show enough work so I can tell what you are doing/thinking.

Part II. Do any **eight** of the problems in this part. If you do more than eight, you will get credit for the best eight. Note that some problems have multiple parts. (15 points each)

1. Let $f(x) = 6x^2 - x^3 - 10$. Find the maximum and minimum values of f and their locations in the interval $[-1, 3]$. In your answer you should clearly indicate which are the values and which are the locations.
2. A gardener wants to enclose an area of 150 square feet in a rectangular plot that is divided in two with a length of fence parallel to one of the sides of the plot. What should the dimensions of the plot be in order to minimize the amount of fence used? How much fence is used?
3. Prove that $\dfrac{d}{dx}(\arcsin x) = \dfrac{1}{\sqrt{1-x^2}}$.

4. Sketch the graph of a differentiable function f with all of the following properties. Indicate on the graph the locations of all local maxima/minima and inflection points. Include in your sketch the tangent line at $x = 1$.

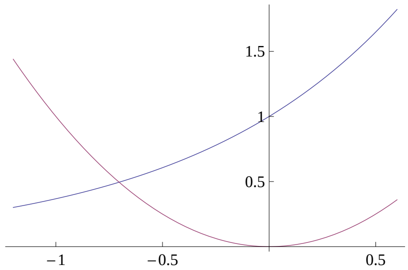
$$f(-2) = f(1) = f(3) = 0 \qquad\qquad f'(1) = -1$$

$$f'(x) > 0 \text{ for } x < -1 \text{ and for } x > 2 \qquad\qquad f'(x) < 0 \text{ for } -1 < x < 2$$

$$f''(x) < 0 \text{ for } x < 0 \qquad\qquad f''(x) > 0 \text{ for } x > 0$$

5. Let $f(x) = x(3 - |x|)$. Determine $f'(0)$. You can't use the product rule since $|x|$ isn't differentiable at $x = 0$, so you have to use the definition of derivative. This isn't as scary as it looks. Use the definition of derivative for $f'(0)$, not for the more general $f'(a)$ or $f'(x)$.

6. The graphs of $y = e^x$ and $y = x^2$ are shown near the point where they cross. Use Newton's Method to approximate the coordinates of the point where they cross (the x-coordinate is the solution of $e^x = x^2$).



(a) What function will you use for Newton's Method? (5 points) $\quad f(x) =$

(b) What is a good first approximation x_1? Why? (2 points)

(c) Find the values of x_2 and x_3 to at least four decimal places. Show enough work so I can tell what you are doing/thinking. (6 points)

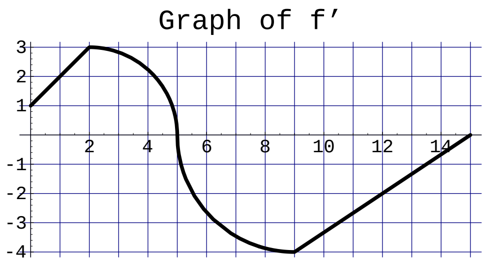
(d) Based on the previous parts, what is the approximate y-coordinate of the crossing point? Be clear how you get this. (2 points)

7. The graph of the derivative of a function f is shown below. Answer the following. **Note: The graph of f is not shown.**

(a) On what interval(s) is f increasing? Briefly indicate how you know.

(b) On what interval(s) is f concave up? Briefly indicate how you know.

(c) At what value of x does f take its absolute maximum value? Briefly indicate how you know.



8. The graph of the derivative of a function f is shown above. The graph consists of two segments and two quarter circles. If $f(0) = -2$, use the Fundamental Theorem of Calculus or the Net Change Theorem to find the exact value of $f(5)$. **Note: The graph of f is not shown.**

9. Suppose f is a continuous function on the interval $[a, b]$. The expressions

$$\int_a^b f(x)\,dx \qquad \text{and} \qquad \int f(x)\,dx\Big|_a^b$$

look similar, but they have very different meanings.

   (a) Describe the meaning of each without using the words "integral" or "integrate." (Think about the context in which we first encountered $\int_a^b f(x)\,dx$. It may help to think about $\int f(x)\,dx|_a^b$ as a two-step process, namely $F(x) = \int f(x)\,dx$ and $F(x)|_a^b$.)

   (b) What does one version of the Fundamental Theorem of Calculus say about how their values are related?

10. Determine if $\int \ln x\,dx$ is equal to $x + x\ln x + C$.

11. Consider $\int_0^2 \sqrt{4 - x^2}\,dx$.

   (a) Compute R_4, the Riemann sum with four subintervals using right endpoints. Use your calculator and give your answer to four decimal places. Show enough work so I can tell what you are doing/thinking. (10 points)

   (b) Is R_4 an overestimate or underestimate of the true value of the integral? Briefly explain. (5 points)

12. Use a substitution to evaluate: $\displaystyle\int \cos(2x)\sqrt{5 + \sin(2x)}\,dx$

Have a good break!

Selected partial answers and hints.

Part I.

1. $-1/3$
2. $y' = \frac{\cos x - 2xy}{2e^{2y}+x^2}$. Be careful not to lose points because of algebra!
3. See what *Mathematica* gets.
4. $y - 10 = -2(x - 5)$. This is the preferred form.
5. $\tan(-\pi/6) = -\sqrt{3}/3$
6. See what *Mathematica* gets.
7. Check by differentiating and seeing that the condition $f(-2) = 0$ holds.
8. $4/3$

Part II.

1. The maximum value is 17, which occurs at $x = 3$.
2. 60 feet of fence are needed.
5. $f'(0) = 2$. The biggest error on this problem is ignoring the absolute value.
6. (c) For x_3 a value between -0.7038 and -0.7036 is good. The value depends on what you use for x_1. (d) About .495.
7. (b) The function is concave up on two intervals. One of them is $[0, 2]$. (c) $x = 5$
8. $f(5) = 2 + 9\pi/4$
10. Think about what $\int f(x)\,dx = F(x) + C$ means.
11. $R_4 = 2.4957$ is an underestimate.
12. See what *Mathematica* gets.