Math 111 Exam 3 Circle section 9:00 11:20 Name:

17 November 2010 100 Points

No *Mathematica*.

"Show enough work to justify your answers."

1. Let $f(x) = \dfrac{3x}{4-5x^2} + x^2\,3^x + \ln 7$. Compute $f'(x)$ and fully simplify. (10 points)

Read Carefully!! Work on at least **six** of the remaining problems. If you work on more than six, you will get credit for the best six. (15 points each)

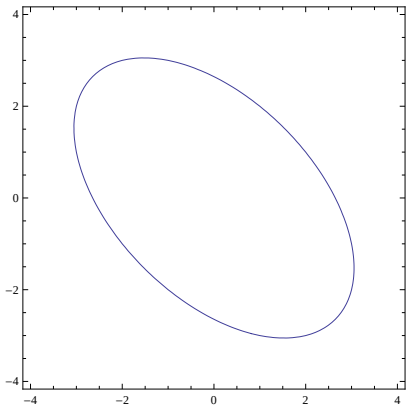
2. Let $f(x) = \sqrt[3]{x}$.
   (a) Find the linear approximation $L(x)$ of $f(x)$ near $x = 8$. (10 points)
   (b) What is $\sqrt[3]{7.8}$ approximately equal to according to this linear approximation? Give your answer to four decimal places. (5 points)

3. Compute the derivative of $f(x) = (3 + \cos x)^{\sin x}$.

4. Suppose that the derivative of a function f is given by $f'(x) = \frac{3}{x^2} - 4x - 5$ and that $f(3) = 7$. Find the formula for $f(x)$.

5. Suppose that $h(x) = f(x^2 + g(x))$, where $f(3) = 6$, $f'(3) = -4$, $g(-2) = -1$, and $g'(-2) = 2$. Compute $h'(-2)$.

6. The graph of $x^2 + xy + y^2 = 7$ is pictured.
   (a) Find the equation of the line tangent to the graph at $(2, 1)$. (10 points)
   (b) Add the line to the picture. (5 points)

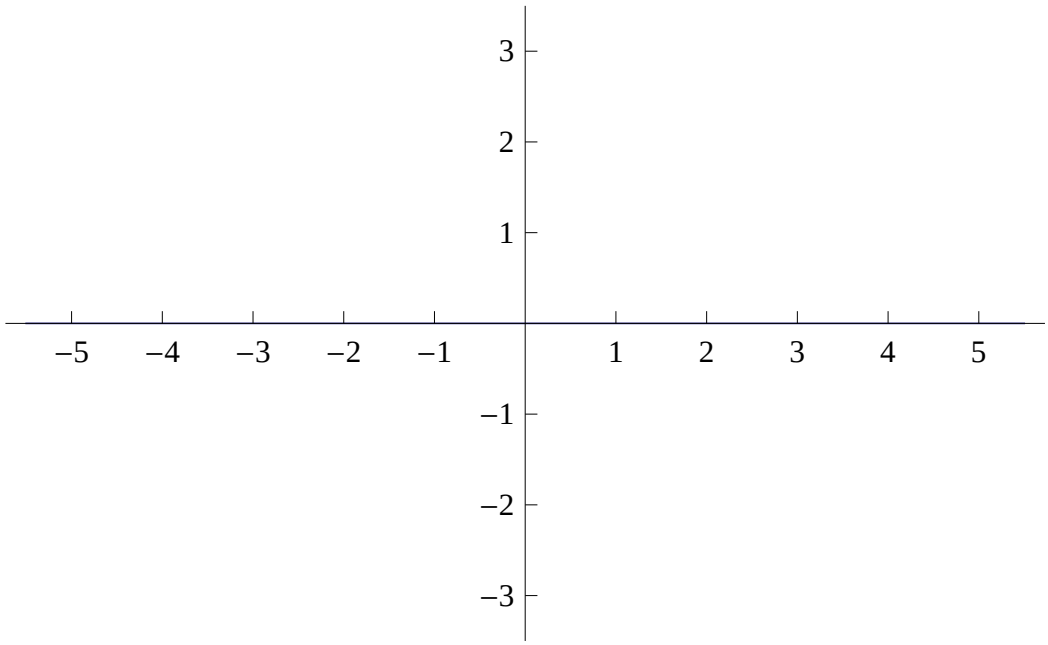


7. Sketch the graph of a continuous function f on $[-5, 5]$ with the following properties. Suggestion: Draw some trial sketches on scratch paper and then draw your final sketch on this page.

$f(2) = -2$ $\quad$ f is differentiable everywhere except at $x = 2$

$f'(x) > 0$ for $-5 < x < 0$ and $2 < x < 5$ $\quad$ $f'(x) < 0$ for $0 < x < 2$

$f''(x) > 0$ for $-5 < x < -2$ $\quad$ $f''(x) < 0$ for $-2 < x < 2$ $\quad$ $f''(x) = 0$ for $2 < x < 5$



8. Professor Krause has discovered a new radioactive compound, which he has named Monon Bellium, or MB. Like all radioactive compounds, it decays at a rate proportional to the amount present. At noon on Monday he had 54 grams of MB. At noon on Wednesday he had 24 grams. How much MB will he have at noon on Thursday?

9. A bug is crawling up a wall at 2 inches per second. A cat is stalking the bug and is moving towards the wall at 6 inches per second. At a particular moment the bug is 4 feet above the floor and the cat is 3 feet from the wall. Determine how fast the distance between them is changing and if the cat is getting closer to the bug or farther from it at that moment. State your answer in a sentence.

Selected answers and hints.

1. See what *Mathematica* gets.

2. $\sqrt[3]{7.8} = f(7.8) \approx L(7.8) = 1.9833$. Note: The goal is to approximate $\sqrt[3]{7.8}$. If your answer doesn't do this, you have made an error.

3. See what *Mathematica* gets. This requires a special differentiation method.

4. $f(x) = -\frac{3}{x} - 2x^2 - 5x + 41$

5. $h'(-2) = 8$. You have to differentiate before you plug in -2.

6. $y - 1 = -\frac{5}{4}(x - 2)$. You can save some time by plugging in the values of x and y immediately after differentiating before solving for y'.

8. 16 grams

9. The cat is getting closer to the bug by 2 inches per second. If you don't get this value, or if you don't see why the cat is getting closer, rather than farther, you have missed an important feature of the velocities.