Math 111 **Exam 2** **Circle section: 9:00 11:20** **Name:**

11 October 2010 100 Points

No calculators or *Mathematica*.

"Show enough work to justify your answers."

1. Let $f(x) = \dfrac{2x}{3x^2 - 7}$. Compute $f'(x)$ and fully simplify. (10 points)

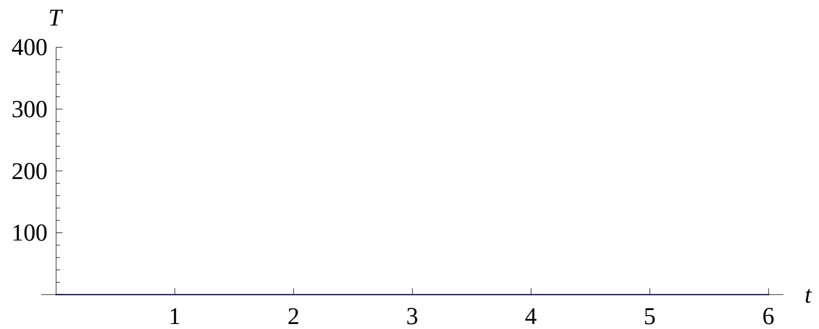
Read Carefully!! Work on at least **six** of the remaining problems. If you work on more than six, you will get credit for the best six. (15 points each)

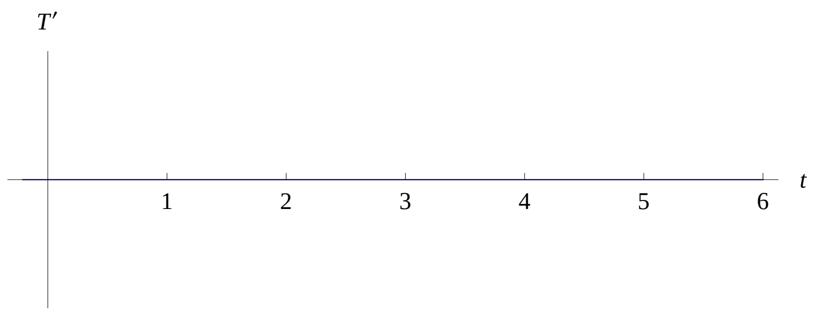
2. Find the point (both coordinates) on the curve $y = \frac{3}{2}x\sqrt[3]{x}$ where the tangent line is parallel to $4x - y = 7$. You do not need to find the equation of the tangent line.

3. Find a function f for which $f'(x) = 6x^2 - x + 3$ and $f(2) = 10$.

4. State the official definition of the derivative and use it to show that $f'(2) = -1/4$, where $f(x) = 1/x$.

5. An oven is at room temperature (70°F) at noon. At some time in the afternoon it is turned on, and it heats up to 350°F in fifteen minutes. After being at that temperature for an hour, it is shut off, and it gradually returns to room temperature. Draw the graphs of T and T', where T is the oven temperature as a function of time in hours after noon. Indicate on each graph where the oven is turned on, where it is turned off, and where it returns to room temperature.





6. Suppose that f is a function for which $f(2\pi/3) = 2\sqrt{3}$ and $f'(2\pi/3) = -10$. Let $g(x) = f(x)\cos x$. Find an equation of the line tangent to the graph of g at the point where $x = 2\pi/3$. For full credit, you must use the point-slope formula and simplify as much as possible. You may leave your answer in point-slope form.

7. A ball is launched from the top of a 50 foot building with an initial upward velocity of 100 ft/sec. The ball's height above the ground t seconds after launch is $s(t) = 100t - 16t^2 + 50$.

   (a) What is the average velocity of the ball during its first four seconds in flight? (5 points)

   (b) What is the instantaneous velocity at $t = 4$? (5 points)

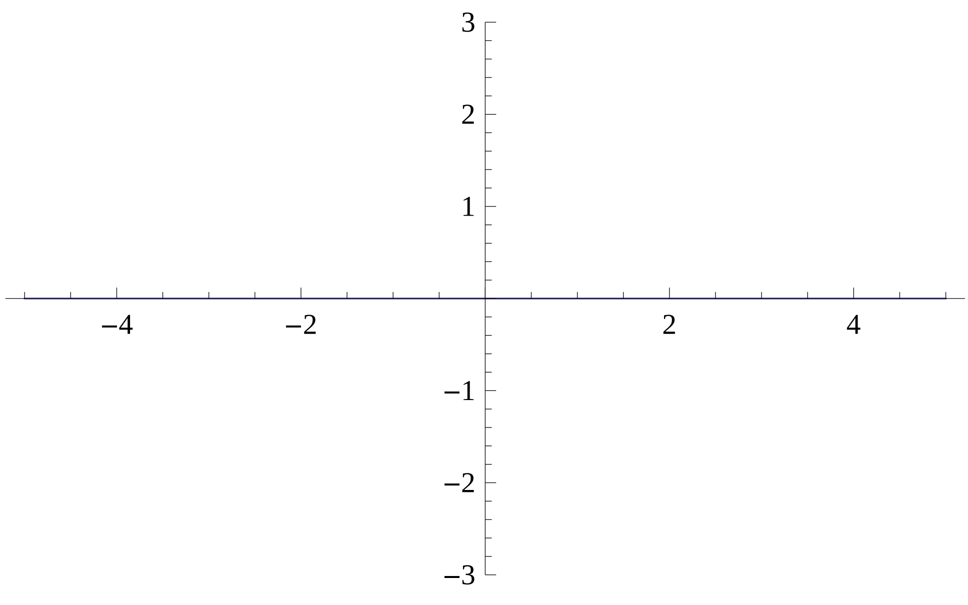
   (c) What is the instantaneous speed at $t = 4$? (5 points)

8. Evaluate $\lim_{x\to 0} \frac{\tan x}{x}$. (If you know L'Hospital's Rule, you may not use it!)

9. Sketch the graph of a function with the following properties. Make the important parts accurate and convincing. Indicate asymptotes with dotted lines.

   $f(x)$ is defined for all x f is continuous everywhere possible

$\lim_{x\to\infty} f(x) = 2$	$f(-1) = -2$	$f(1) = -1$	$f(3) = 0$
$\lim_{x\to-\infty} f(x) = 1$	$f'(-1) = 0$	$\lim_{x\to 1^+} f(x) = \infty$	$f'(3) = 1$



10. Let $f(x) = x^4 - x^3 - 2x + 1$.

    (a) Carefully explain why f must have a zero somewhere between $x = 0$ and $x = 2$. (Suggestion: Evaluate f at a few places.) (10 points)

    (b) What is the name of the theorem you are using? (5 points)

Selected answers and hints.

1. Leave the bottom factored. If something is going to cancel from the top and bottom, you won't know it if you multiply the bottom out.
2. Don't use the product rule! Ans: $(8, 24)$
4. The official definition is not words about slope, it's one of the formulas with a limit.
5. It's okay for the graph of T to have corners, in which case the graph of T' will have jump discontinuities. Note that $T' = 0$ between the times when the oven is fully heated and when it is shut off.
6. $y + \sqrt{3} = 2(x - 2\pi/3)$
7. (a) 36 ft/sec, Note: Average velocity does not involve a derivative. (c) 28 ft/sec
8. 1
9. Be careful that what you draw satisfies the Vertical Line Test. Contrary to popular belief, the graph of a function can cross a horizontal asymptote. It cannot, however, cross a vertical asymptote (but it can have a point on a vertical asymptote).
10. Note that $f(0) = 1$, $f(1) = -1$, and $f(2) = 5$. A function has a zero when its graph crosses the x-axis. Use the Intermediate Value Theorem to draw your conclusion.