Math 111 **Exam 3** **Name:**

16 November 2007 100 Points

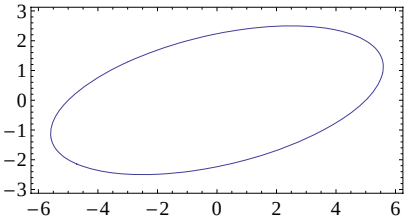
No calculators or *Mathematica*.

"Show enough work to justify your answers."

1. You must do this problem (10 points). Evaluate: $\displaystyle\int_1^3 \left(x^2 + \frac{1}{x}\right) dx$

READ CAREFULLY! Do any **six** of the remaining problems. If you work on more than six you will get credit for the best six. I suggest reading through all of them first. Exact answers are expected unless otherwise indicated. (15 points each)

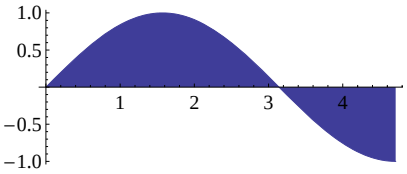
2. The curve $x^2 - 2xy + 5y^2 = 25$ is pictured. Find both points on the curve where the tangent line is horizontal.



3. Let $f(x) = \sin x$, pictured below.

   (a) Find the *signed* area bounded by the graph of f and the x-axis on the interval $[0, 3\pi/2]$.

   (b) Find the *actual* area bounded by the graph of f and the x-axis on the interval $[0, 3\pi/2]$.



4. Give the details of the computation showing that $\dfrac{d}{dx}(\arccos x) = -\dfrac{1}{\sqrt{1-x^2}}$.

5. Sketch the graph of a continuous function f defined for $x > -2$ with all of the following properties. Be sure to scale the axes in a manner appropriate to the data.

$$\lim_{x\to -2^+} f(x) = \infty, \qquad \lim_{x\to\infty} f(x) = 3, \qquad f''(x) > 0 \text{ for } x < 4, \qquad f''(x) < 0 \text{ for } x > 4$$

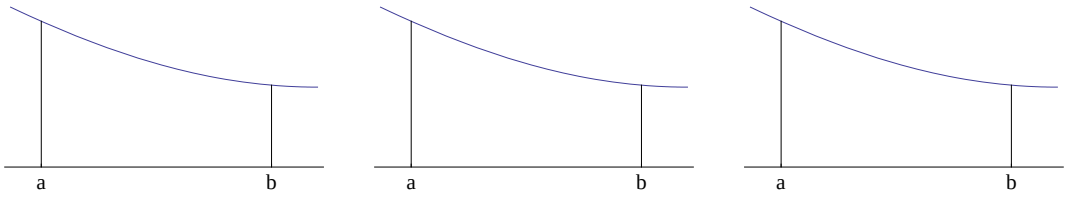
6. Evaluate $\int x^2\sqrt{x^3+10}\,dx$.

7. Suppose $h(x) = f(x^2 g(x))$, where f and g are differentiable. Given $f(2) = 8$, $f'(2) = -5$, $f(3) = 4$, $f'(3) = 5$, $g(-1) = 3$, and $g'(-1) = 2$, find the equation of the line tangent to the graph of h at $x = -1$.

8. Determine if the following is true: $\int \ln x\,dx = x\ln x + x + C$.

9. Consider the integral $\int_a^b f(x)\,dx$, where three copies of the graph of f are shown.

   (a) Assuming three subdivisions of the interval $[a, b]$ are used, draw the pictures illustrating the left, right, and trapezoid approximating sums for the integral.



Left sum Right sum Trapezoid sum

   (b) Which of these sums overestimate and which underestimate the value of the integral? Which gives the closest estimate? Explain.

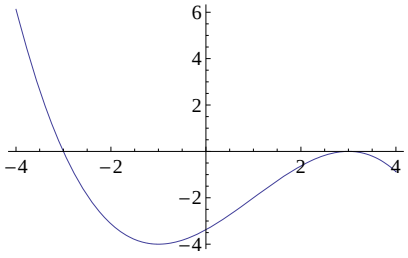
10. Let f be the function graphed below. Let $F(x) = \int_{-3}^{x} f(t)\,dt$. Answer the following. Detailed reasons are not necessary.

    (a) For what values of x in $[-4, 4]$ is $F(x)$ positive?

    (b) Where is F increasing?

    (c) Where is F concave up?

    (d) Where does F have local maxima (don't forget the endpoints)?



Hints and partial answers.

1. See what *Mathematica* gets. Note: $\ln 5 - \ln 3$ is *not* equal to $\ln 2$!

2. You don't need to solve for y'. One of the points is $(5/2, 5/2)$.

3. The signed area is 1. The actual area is 3.

5. A curve can cross its asymptote.

6. See what *Mathematica* gets. At least *think* about taking the derivative of your result to check it!

7. $y - 4 = -20(x + 1)$

8. Hint: Verbalize what the equation means without using the words "antiderivative" or "integral."

9. The trapezoid sum overestimates and gives the best approximation.

10. The pictured graph is the graph of f, not the graph of F. Remembering that $f = F'$ makes this a problem from Chapter 1. Part (d) is asking for an x-value, not for a point on the graph.