

8 October 2007

100 Points

No calculators or *Mathematica*.*“Show enough work to justify your answers.”*

1. What is the derivative of $f(x) = \frac{3}{\sqrt{x}} - 2 \cos x + \ln x + \sqrt{\pi}$? (10 points)
2. Suppose that the derivative of a function f is given by $f'(x) = 2e^x + 3 \cos x + x^3$ and that $f(0) = 6$. Find the formula for $f(x)$. (15 points)

Read Carefully!! There are seven more problems, each worth fifteen points. Work on any **five** of them. If you work on more than five, you will get credit for the best five.

3. The temperature of a hot object decreases at a rate proportional to the difference between the temperature of the object and the temperature of its surroundings. A cup of hot coffee is cooling in a 70° room. Let $T(t)$ denote the temperature of the coffee as a function of time. Write a differential equation that relates $T(t)$ and $T'(t)$. (Note: There will not be much partial credit given on this problem, so if you are unsure of your answer, you should think of this as an extra problem.)
4. Determine if $y(x) = x^2$ is a solution of the differential equation $xy'' + 1 = 2y' + \sqrt{y}$. Assume $x > 0$. Be sure to state your conclusion.
5. Let $f(x) = x^2 - 3x + 2$. Use the definition of derivative to find $f'(4)$.
6. Evaluate $\lim_{x \rightarrow 3} \frac{x - 3}{x^2 - x - 6}$.
7. Determine values of a , b , and c so that the following function f is continuous and differentiable at $x = 0$, and so that $f(4) = 0$.

$$f(x) = \begin{cases} x + 4, & \text{if } x < 0 \\ ax^2 + bx + c, & \text{if } x \geq 0 \end{cases}$$

8. The general solution of the DE $y' = k(y - b)$ is $y = b + Ae^{kx}$, where A is an arbitrary constant. Find the solution of the IVP $y' = 2(y - 10)$, $y(0) = 50$.
9. Find the the maximum and minimum values of $f(x) = x^3 - 27x + 5$ and their locations on the interval $[-4, 1]$. Do your computations below, and then put your answers in the blanks.

The maximum value is _____, and it occurs at _____.

The minimum value is _____, and it occurs at _____.

Selected answers and hints.

1. Be careful when you take the derivative of $\sqrt{\pi}$!
3. $T' = k(T - 70)$
4. y is not a solution of the differential equation.
5. Remember the definition of the derivative involves a limit.
7. $a = -1/2$, $b = 1$, $c = 4$
8. $y = 10 + 40e^{2x}$
9. The maximum value is 59. The minimum value occurs at 1.