Math 111 **Exam 1** **Name:**

17 September 2007 100 Points

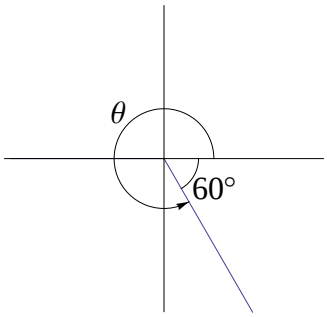
No calculators or *Mathematica*, except as indicated.

"Show enough work to justify your answers."

1. Consider the following statement: *"House prices are still going down, but not as fast as they were."* Let $P(t)$ denote the average price of a house as a function of time over the time period suggested by the statement. Circle the conclusions that can be made from this statement. No justifications are necessary. Note: You should circle **all** the conclusions that can be made, and **only** those that can be made. (10 points)

P is	positive	negative	increasing	decreasing	concave up	concave down
P' is	positive	negative	increasing	decreasing	concave up	concave down
P'' is	positive	negative	increasing	decreasing	concave up	concave down

2. Let θ be the pictured angle.



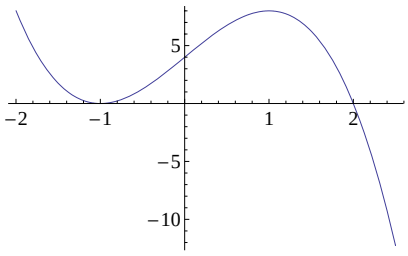
   (a) What is the radian measure of θ? (4 points)

   (b) What are the exact values of $\sin\theta$ and $\tan\theta$? (6 points)

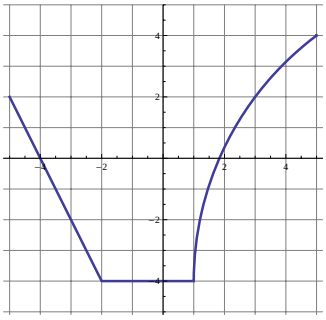
3. Solve each equation for x. (10 points)

   (a) $2^{3x-2} = 5$ (b) $\log_{10}(x + 1000) = 2$

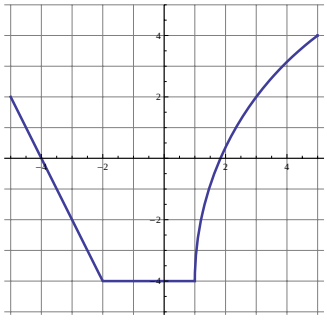
4. The graph of a cubic polynomial f is pictured. What is the formula of f? Two important things to note: 1) The directions the graph goes for large positive and negative values of x, and 2) The value of $f(0)$. You may leave your answer in factored form. You may use *Mathematica* or a graphing calculator, however this problem can be done without either. (10 points)



5. Two copies of the graph of a function f are shown. Define functions g and h by $g(x) = f(x) + 2$ and $h(x) = f(x + 2)$. Draw the graph of g on the left graph and the graph of h on the right graph. (10 points)



$g(x) = f(x) + 2$



$h(x) = f(x + 2)$

6. What is the natural domain of $f(x) = \sqrt{25 - x^2}$? (10 points)

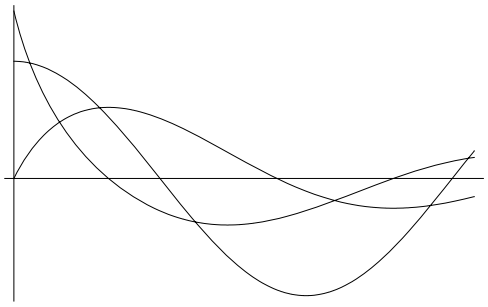
7. Values of a function f are given in the following table.

x	1	1.1	1.2	1.3	1.4
$f(x)$	2	1.65	1.39	1.18	1.02

   (a) Give estimates for $f'(1.1)$ and $f'(1.3)$. For full credit you must write down enough so I can tell what you are thinking. You may use your calculator to do arithmetic. (6 points)

   (b) Based on the evidence, is the graph of f concave up or concave down? Briefly explain. (4 points)

8. The graphs of three functions are shown, f, f', and an unrelated function g. Clearly indicate (with labels and arrows) which graph is for which function. Briefly explain. (10 points)



9. The following questions concern a function f defined for $-4 \leq x \leq 5$ whose derivative is shown below. **The graph of f is not shown.** (20 points)

   (a) Does f have a local maximum? If so, at what x-value? Explain. (5 points)

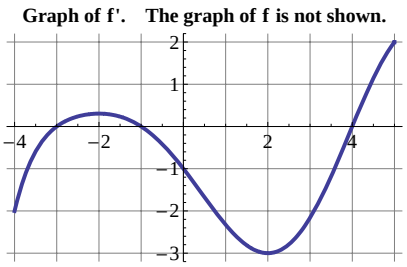
   (b) Where is f concave down? How do you know? (5 points)

   (c) Suppose $f(5) = -3$. Find an equation for the line tangent to the graph at $x = 5$. Put your equation in point-slope form. (4 points)

   (d) In each blank, write integers that make both inequalities true. Your integers should be as close together as possible.

   $\_\_\_\_\_ \leq f'(x) \leq \_\_\_\_\_$ for $1 \leq x \leq 3$ (1 point)

   (e) Suppose $f(1) = 4$. Use your answer from the previous part and the Speed Limit Law to explain why $-2 \leq f(3) \leq 0$. (5 points)



This is the graph of f'.
The graph of f is not shown.

Selected partial answers and hints.

1. Only one conclusion can be made about P'', two about P', and three about P. P is positive, decreasing, and concave up.

2. θ is a multiple of $\pi/3$, and the triangle involved is a 30-60-90 triangle. When you are outside of the first quadrant, it is best not to think of the trig functions in terms of opposite, adjacent, and hypothenuse.

3. To get the exponent down on the main line, take some kind of log of both sides. To get rid of the log, exponentiate both sides.

4. A common mistake is to take $f(x) = (x^2-1)(x-2)$ as a first guess. This has $f(1) = 0$, and so it has an undesired root. To get the y-intercept to be 4, compute $f(0)$. If $f(0)$ is not 4, compensate by adjusting the function. Don't compensate by adding something, because that changes the roots.

5. To get the graph of g, shift the graph of f up 2 units. To get the graph of h, shift the graph of f to the *left* 2 units. If this seems backwards, you need to review Section 1.2.

6. $[-5, 5]$ or $-5 \leq x \leq 5$. The endpoints are included in this case. Note: It is not sufficient to simply see where the function is 0, and then to conclude that the domain is between the zeros. If you did this, then you would conclude that $\sqrt{x^2-25}$ has the same domain, which it doesn't!

7. (a) There are three reasonable answers for each, but the best ones are $f'(1.1) \approx -3.05$ and $f'(1.3) \approx -1.85$.

   (b) Concave up. The best way to see this is to note that f' is increasing from part (a). Alternatively, you can plot the data points to see what the graph might look like, but you need to plot a reasonably sized graph. Too many people draw a graph that would fit on a postage stamp and then try to draw conclusions from it!

8. The graph that goes through the origin is f. The one that has the largest y-intercept is f'.

9. The biggest mistake on this problem was to answer the questions about the pictured graph. **The graph of f is not shown. THE GRAPH OF f IS NOT SHOWN!!**

   (a) f actually has *three* local maxima. They are at $x = -4$, $x = -1$, and $x = 5$.

   (b) $[-2, 2]$

   (c) $y + 3 = 2(x - 5)$

   (d) $-3 \leq f'(x) \leq -2$. Note: $-2 \leq f'(x) \leq -3$ would imply that $-2 \leq -3$(!).

   (e) The Speed Limit Law implies that $-3(3-1) \leq f(3) - f(1) \leq -2(3-1)$. Simplify this and plug in $f(1) = 4$.