

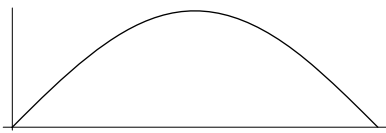
13 November 2006

100 Points

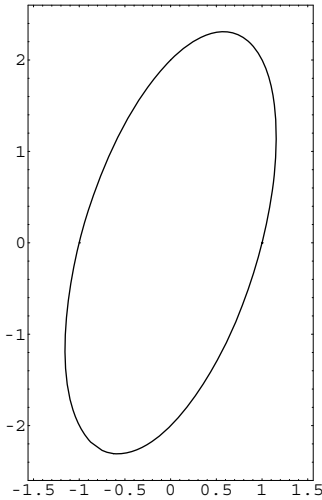
No calculators or *Mathematica*.*“Show enough work to justify your answers.”*

READ CAREFULLY! There are eight problems. Do any **five** of them. If you work on more than five you will get credit for the best five. I suggest reading through the entire exam before you start on any of the problems. Exact answers are expected unless otherwise indicated. (20 points each)

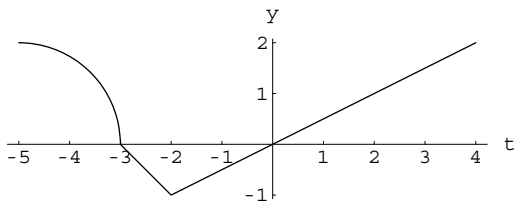
1. Find the area under one arch of $y = \sin x$.



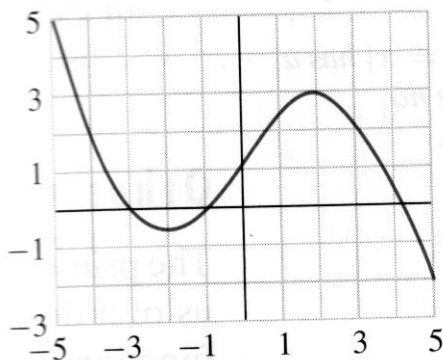
2. What are the two values of θ between $-\pi$ and π for which $\sin \theta = -1/2$? Which of these values is $\arcsin(-1/2)$? (For the second question, it may help to think about the graph of $\arcsin x$. If you don't remember the graph, think about the graph of $\sin x$ and what you do to it to get the graph of $\arcsin x$.)
3. Let $y(t) = e^{-t} \sin t$. Determine if y satisfies the differential equation $y'' + 2y' + 2y = 0$.
4. Give the details of the computation showing that $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$.
5. The ellipse $4x^2 - 2xy + y^2 = 4$ is pictured.
- Verify that the point $(1, 2)$ is on the curve. (3 points)
 - Find an equation of the line that is tangent to the curve at this point. Note: To simplify the computation, it's okay to plug in the values before doing the algebra. For full credit use the point-slope formula. (14 points)
 - Add the tangent line to the picture. (3 points)



6. Evaluate $\int \tan x \, dx$. Hint: Write $\tan x$ in terms of $\sin x$ and $\cos x$ and use a substitution.
7. The graph of a function f is shown. The graph consists of line segments and a quarter circle. Let $G(x) = \int_{-5}^x f(t) \, dt$, and let $F(x) = \int_0^x f(t) \, dt$.



- (a) What is the value of $G(0)$? Put your answer to the right of the graph. (7 points)
- (b) Give the formula for $F(x)$ for $x \geq 0$. (7 points)
- (c) Give the formula for $G(x)$ for $x \geq 0$. (6 points)
8. Consider $\int_{-5}^0 f(x) \, dx$, where the graph of the function f is shown. Estimate the value of the integral to the nearest integer or between two consecutive integers. Briefly explain. Shade the area representing the integral.



Selected answers and hints.

1. The area is 2. Be sure to subtract the right way: $F(x)|_a^b$ means $F(b) - F(a)$, not $F(a) - F(b)$.
2. $\arcsin(-1/2) = -\pi/6$
5. (b) Tangent line: $y - 2 = -2(x - 1)$. Point-slope form is preferred.
6. $-\ln |\cos x| + C$
7. (c) $G(x) = \pi - \frac{3}{2} + \frac{x^2}{4}$
8. The value is between 4 and 5, but it is closer to 5.