

16 October 2006

100 Points

No calculators or *Mathematica*.*“Show enough work to justify your answers.”*

1. What is the derivative of $h(t) = \sqrt[5]{t^2} + \frac{5}{t^4} + \pi^7$? (10 points)
2. Suppose that the derivative of a function f is given by $f'(x) = \frac{3}{x^2} - 4x + 5$ and that $f(3) = 7$. Find the formula for $f(x)$. (15 points)

Read Carefully!! There are seven more problems, each worth fifteen points. Work on any **five** of them. If you work on more than five, you will get credit for the best five.

3. An infectious case of ambivalentitus is spreading through the Wabash student body. The rate at which the number of infected students increases per day is proportional to product of the number of students infected and the number of students not yet infected. Let $S(t)$ denote the number of infected students as a function of time in days. Assuming there are 850 students, write a differential equation that relates $S(t)$ and $S'(t)$.
4. Determine if $y(x) = 3 \ln x + 5 + x^2$ is a solution of the differential equation $x^2 y'' + xy' = x^2 + 3$. Be sure to state your conclusion.
5. Let $f(x) = \frac{5 - 2x^2}{x^2 - 8x + 16}$. Evaluate all three of the following limits. Then comment on what they imply about the graph of f . Be specific.
 - (a) $\lim_{x \rightarrow 4} f(x)$
 - (b) $\lim_{x \rightarrow \infty} f(x)$
 - (c) $\lim_{x \rightarrow -\infty} f(x)$
6. Let $f(x) = \sqrt{x}$. Use the definition of derivative to find $f'(4)$.
7. Find the equation of the line tangent to the graph of $f(x) = x^3 - 6x^2$ at its inflection point. (Start by finding the inflection point.) For full credit, you must use the point-slope formula.
8. Find values for the constants A and B such that the function $y(x) = A \cos x + B \sin x$ satisfies $y(5\pi/6) = 0$ and $y'(5\pi/6) = 1$.
9. A square piece of cardboard is 6 inches on a side. Equal squares are to be cut out of the corners and the sides folded up to form a box open at the top. How big should the squares be that are cut out in order to maximize the volume of the box, and what is the maximal volume? Be sure to answer the question, and be clear how you know you have found a maximum, as opposed to a minimum. (Note: If the squares cut out have side length x , then the resulting volume is $V(x) = x(6 - 2x)^2$.)

Selected partial answers and hints.

3. $S'(t) = kS(t)(850 - S(t))$

4. It is not a solution. For full credit it needs to be clear from what you write that $x^2y'' + xy'$ and $x^2 + 3$ are not equal.

5. (a) $-\infty$ (b) -2 (c) -2

For full credit you need to show the algebra that leads to your conclusion. As a result of (a), the graph has a vertical asymptote at $x = 4$ with the graph going down on both sides of the asymptote. As a result of (b) and (c), the graph has $y = -2$ as a horizontal asymptote in both directions.

6. Check your answer using the power rule.

7. $y + 16 = -12(x - 2)$

8. $A = -1/2$, $B = -\sqrt{3}/2$

9. To maximize the volume, you should cut out 1"-by-1" squares. The maximum volume is 16 cubic inches. To be sure you have found the maximum (as opposed to a relative maximum or a minimum), you need to check the endpoints (best and easiest) or use the second derivative test.