Math 111   **Exam 1**   **Name:**

18 September 2006   100 Points
No calculators or *Mathematica*, except as indicated.
"Show enough work to justify your answers."

1. (15 points) Consider the following statement: *"The child's temperature has been rising for the last two hours, but not as rapidly since we gave her the antibiotic an hour ago."*

   Let $T(t)$ denote the child's temperature as a function of time in hours over the time period suggested by the statement.

   (a) Sketch a graph of T above that is consistent with the statement. Let $t = 0$ correspond to the time when the medicine was given. Indicate the times $t = -1$, 0, 1 on the t axis.

   (b) Is $T'(t)$ positive or negative on any portion of the interval $[-1, 1]$? If so, indicate which portion and why the statement implies this.

   (c) Is $T''(t)$ positive or negative on any portion of the interval $[-1, 1]$? If so, indicate which portion and why the statement implies this.
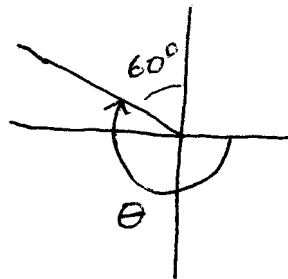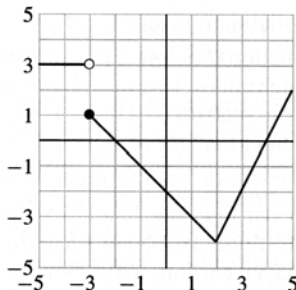

2. Let θ be the pictured angle.

   (a) What is the radian measure of θ? (4 points)

   (b) What are the exact values of $\sin \theta$ and $\tan \theta$? (Exact values are always expected in a math class unless indicated otherwise.) (6 points)

3. Solve each equation for x. (10 points)
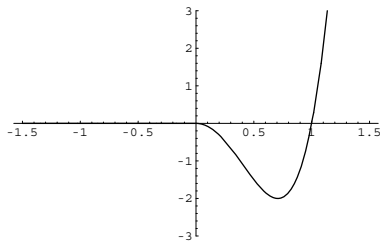
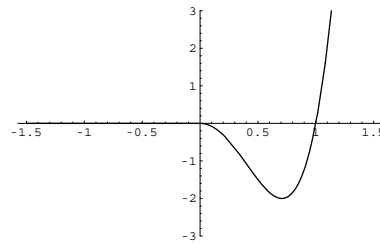   (a) $5^{2+x} = 15$ $\log_6(4x) = 2$

4. Give the piecewise formula for the pictured function. (10 points)

5. Two copies of a partial graph of a function f are shown. On the left graph, extend the graph of f assuming f is an even function. On the right graph, extend the graph of f assuming f is an odd function. (10 points)

Even extension   Odd extension

6. What is the natural domain of $f(x) = \dfrac{1}{\sqrt{9 - x^2}}$? (10 points)

7. On the computer go to our course folder: Courses on Caleb N:/Math/Math111/Foote. Open the *Mathematica* file Exam1.nb. A function f is defined there. Its graph will be drawn when you evaluate the first input cell. You are to give a *good* approximation of $f'(1)$. Do this by using two x-values as inputs and their corresponding outputs. Use *Mathematica* to evaluate the function and do the arithmetic. Write your answers below. Round your answers off to three decimal places. It is in your interest to give me some idea of how you are thinking about $f'(1)$ so I can give some partial credit if your numbers aren't correct. (The *Mathematica* file defined the function and allowed the user to find values of it.) (10 points)

The two x-values you used:

The corresponding values of $f(x)$:

Approximate value of $f'(1)$:

8. The graphs of f, f', and f'' appear below. Determine which is which, and briefly explain how you know. (10 points)
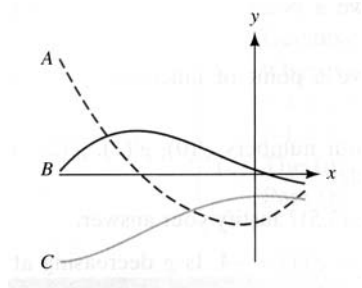
9. The following questions concern a function f whose derivative is shown below. **The graph of f is not shown.**

    (a) Where (at what x-value) does f take its minimum on the interval $[-4, 4]$? Explain. (4 points)

    (b) Assume that $f(-2) = 1$. Give the equation of the line tangent to the graph of f at $x = -2$ in point-slope form. (4 points)

    (c) What are the best lower and upper estimates for $f'(x)$ on the interval $[-2, 0]$, that is, what are the closest numbers L and U such that $L \leq f'(x) \leq U$ for $-2 \leq x \leq 0$? (2 points)

    (d) Assume that $f(-2) = 1$. Use the numbers L and U from the previous part and the Speed Limit Law to give lower and upper estimates for $f(0)$. (5 points)
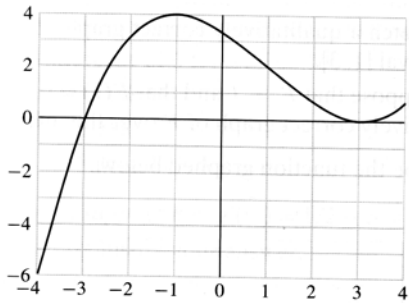
This is the graph of f'. **The graph of f is not shown.**

Partial answers and hints.

1. In parts (b) and (c), be sure to indicate why/how the words in the statement imply your conclusion. Don't simply talk about the graph you draw.

2. Learn how to do the standard angles in radians *without* converting to degrees. It's easier, you are less apt to make a mistake, and you will become more comfortable with radians.

3. (a) $x = \frac{\ln 15}{\ln 5} - 2$ or $x = \log_5 15 - 2$. An approximate answer is not as good as an exact one. You can give the approximation along with the exact answer, but not instead of it. The exceptions to this are when an approximation is asked for, or when the exact answer is very unwieldy.

5. One is symmetric about the y-axis, the other is symmetric about the origin.

6. $-3 < x < 3$ or $(-3, 3)$

8. $f = B$, $f' = A$, $f'' = C$

9. (a) The minimum occurs at $x = -3$. Note: The answer is an x-value, not a point.
    (b) $y - 1 = 3(x + 2)$
    (c) $3 \leq f'(x) \leq 4$
    (d) $7 \leq f(0) \leq 9$