Math 111 **Final Exam** **Name:**

13 December 2005 200 Points

"Show enough work to justify your answers."

READ CAREFULLY! This exam has two parts. In Part I you are to work all of the problems, and you should write your answers on the exam. In Part II you are to work at least eight of the problems, and you are to work each problem on a separate sheet of blank paper.

Part I. Do **all eight** of the problems in this part. Show all steps of every computation. Your answers must be independent of *Mathematica*. (10 points each)

1. State the definition of $f'(a)$ and use it to compute $f'(2)$ when $f(x) = 2x^2 - 3x$.

2. Let $f(x) = x^3 - 3x^2$. Find the maximum and minimum values of f and their locations in the interval $[1, 4]$. In your answer you should clearly indicate which are the values and which are the locations.

3. Differentiate and simplify: $f(x) = \dfrac{x^2}{x^3 - 5} - \ln x \, \sin x + \sqrt[3]{x^5 + x}$

4. Evaluate: $\displaystyle\int_{-1}^{3} (x^2 - x + 2)\, dx$

5. Find the formula for $f(x)$ where $f'(x) = 6x^2 - 4x + 3$ and $f(2) = -4$.

6. Evaluate the following using the substitution $u = 2x + 1$: $\displaystyle\int x(2x+1)^{10}\, dx$

7. Evaluate the following: $\displaystyle\lim_{x\to\infty} \frac{1 + 2x - 3x^2}{2x^2 + x - 7}$

8. Given $e^{2x} + y^2 = x \cos y$, compute dy/dx.

Part II. Do any **eight** of the problems in this part. If you do more than eight, you will get credit for the best eight. Note that some problems have multiple parts. You may use *Mathematica* in your solutions, except as indicated. If you use *Mathematica*, be very clear what you use it for, and show enough work that I can tell what you are thinking. If you have any question about what constitutes sufficient work, be sure to ask. If you have trouble with *Mathematica*, be sure to ask for help. (15 points each)

$\Longrightarrow$ Please do each problem on a **separate** sheet of paper.

$\Longrightarrow$ Put the problem number in the upper **right** corner of the page.

$\Longrightarrow$ **Avoid** writing anything in the upper left corner (to leave room for a staple).

9. Sketch the graph of a differentiable function f with all of the following properties. Be sure to make the size of your graph and the scale on the axes commensurate with the data. Indicate on the graph the locations of all local maxima/minima and inflection points.

$$f(-2) = f(1) = f(3) = 0$$

$$f'(x) > 0 \text{ for } x < -1 \text{ and for } x > 2 \qquad f'(x) < 0 \text{ for } -1 < x < 2$$

$$f''(x) < 0 \text{ for } x < 0 \qquad f''(x) > 0 \text{ for } x > 0$$

10. Suppose that f is a function, that $f(8) = 4$, and that $2 \leq f'(x) \leq 3$ for all x. Give upper and lower estimates for $f(3)$. Explain. For full credit, your upper and lower estimates must be as close to each other as the information allows.

11. Evaluate by hand, showing all steps (no *Mathematica*): $\displaystyle\int \frac{2-x}{1+x^2}\,dx$

12. A gardener is laying out a rectangular garden along a long, straight wall. No fence is needed along the wall (see picture on board). If she has 100 feet of fence, what are the dimensions and area of the garden with the largest area?

13. A tank of water is draining from a hole in the bottom of the tank. Torricelli's Law says that the depth of the water changes at a rate proportional to the square root of the depth.

    (a) If $D(t)$ is the depth of the water at time t, write a differential equation that expresses this law.

    (b) Show that $D(t) = \left(\dfrac{kt}{2} + C\right)^2$ is a solution of your differential equation. Here C is an arbitrary constant and k is a constant that should appear in your differential equation.

14. See what *Mathematica* gets for $\int \arctan x\,dx$ and verify it by hand. Note: In *Mathematica* the function is written as ArcTan[x].

15. The following table gives values of a function f for some values of its input.

x	0	0.5	1.	1.5	2
$f(x)$	0	1.8	3.1	3.6	4.2

(a) Use the data to give an approximation for $f'(1.25)$. Show enough work so I can tell what you are thinking. (5 points)

(b) Use L_3 to approximate $\int_{1/2}^{2} f(x)\,dx$. Show enough work so I can tell what you are thinking. (7 points)

(c) Do you think that your approximation overestimates or underestimates the true value of the integral? What feature of the function causes this? (Possible features: increasing, decreasing, concave up, concave down.) (3 points)

16. A radar speed detector is set up on a bridge 30 feet above a highway. A car is on the highway moving towards the bridge. At the moment the car is 40 feet from the base of the bridge, the radar (aimed at the car) gives a reading of 50 miles per hour. How fast is the car moving? (Note: You do not need to convert from feet to miles.)

17. Find a number $b > 1$ such that the area under $f(x) = 1/\sqrt{x}$ between 1 and b is 10 square units.

18. Use a suitable quadratic approximation of $f(x) = \sqrt[3]{x}$ to approximate $\sqrt[3]{29}$.

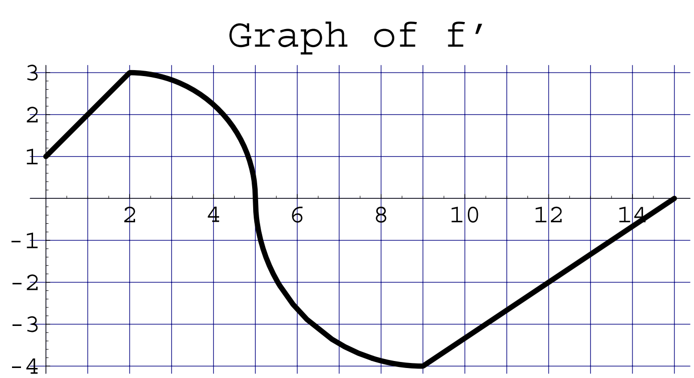
19. Explain how you can know that $f(x) = x - \cos x$ has a root (or zero) in the interval $[0, \pi]$ even if you don't know what the graph looks like. What theorem are you using?

20. The graph of the derivative of a function f is shown. Note that the graph consists of two segments and two quarter circles.
Answer the following. **Note: The graph of f is not shown.** (5 points each)

(a) At what values of x does f take its absolute maximum and minimum values on $[0, 15]$? Briefly indicate how you know.

(b) If $f(5) = 2$, what is the exact value of $f(0)$? Note: Since we have done integration, we have a precise way of answering this.

(c) Does the graph of f lie above or below its tangent line at $x = 6$? How do you know?



Have a good break!

Selected partial answers and hints.

2. The maximum value is 16, which occurs at 4. The minimum value is -4, which occurs at 2.

8. $dy/dx = \dfrac{\cos y - 2e^{2x}}{2y + x\sin y}$

10. $-11 \le f(3) \le -6$

12. The largest area is 1250 square feet (but this isn't the answer).

15. $L_3 = 4.25$. This is an underestimate.

16. 62.5 miles per hour

17. $b = 36$

18. $\sqrt[3]{29} \approx 3.072245$

19. Use the Intermediate Value Theorem.

20. (a) The maximum occurs at 5; the minimum at 15. (b) $f(0) = -2 - 9\pi/4$ (c) below