

16 November 2005

100 Points

No calculators or *Mathematica*, except as indicated.*“Show enough work to justify your answers.”*

READ CAREFULLY! There are eight problems. Do any **five** of them. If you work on more than five you will get credit for the best five. I suggest reading through the entire exam before you start on any of the problems. Exact answers expected unless otherwise indicated. *Mathematica* may be used only in the last problem. (20 points each)

1. Evaluate $\int \frac{1+x}{1+x^2} dx$.

Hint: Break it up into two parts and use the substitution $u = 1 + x^2$ on one of them.

2. Verify that $\int \cos \sqrt{x} dx = 2 \cos \sqrt{x} + 2\sqrt{x} \sin \sqrt{x} + C$.

Hint: What does an equation like this mean?

3. The pictured curve is the graph of $3(x^2 + y^2)^2 = 25(x^2 - y^2)$. (This curve is called a lemniscate.)

(a) Verify that the point $(2, 1)$ is on the curve.

(b) Find an equation of the line that is tangent to the curve at this point. Note: To simplify the computation, it's okay to plug in the values before doing the algebra.

(c) Add the tangent line to the picture.

4. Give the details of the computation showing that $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$.

5. The graph of a function f is shown (OZ, second edition, pg 321, upper left). The graph consists of line segments and a semicircle. Let $G(x) = \int_{-5}^x f(t) dt$.

(a) Find the equation of the line tangent to the graph of G at $x = 1$.

(b) Is the graph of G concave up, concave down, or neither at $x = 1$? Explain.

6. The graph of $y = \sqrt{x} - 2$ is shown. Determine the amount of area shaded. Note: You are to determine the actual area, not the signed area. (The graph is for $0 \leq x \leq 9$. The area shaded is that between the graph and the x-axis.)

7. Sketch the graph of a continuous function f defined for $x > 0$ such that all of the following hold:

a) $\lim_{x \rightarrow \infty} f(x) = 3$, b) $\lim_{x \rightarrow 0^+} f(x) = \infty$, c) $f''(x) < 0$ for $x > 5$

8. This problem has to do with approximating the integral $\int_1^3 1.1^{x^2} dx$ with the R_4 approximation.

(a) The pictured curve is the graph of $f(x) = 1.1^{x^2}$. Add the rectangles for the approximation to the picture.

(b) Find the areas of the rectangles and indicate them on the picture. Give your answers to three decimal places. There is a *Mathematica* notebook Exam3.nb in the Math111\Foote folder that you may use if you like. It has the function already defined. You may also use your calculator.

(c) Based on the previous part, what approximation do you get for the integral?

$$\int_1^3 1.1^{x^2} dx \approx$$

Selected partial answers and hints.

1. $\arctan x + \frac{1}{2} \ln(1 + x^2) + C$

2. The hint says it all!

3. There is no reason to multiply the equation out. Doing so only makes the computations worse. The slope is $-2/11$.

5. (a) $y = 3 - 2\pi$ (b) Concave up

6. The area is $16/3$. (The signed area is 0, but that was not asked for.)

8.(c) $\int_1^3 1.1^{x^2} dx \approx R_4 = 3.438$