Math 111 **Exam 2** **Name:**

17 October 2005 100 Points

No calculators or *Mathematica*.

"Show enough work to justify your answers."

1. What is the derivative of $g(y) = \sqrt[3]{y^5} + 2\ln y$? (10 points)

2. Suppose that the derivative of a function h is given by $h'(t) = e^t + 2\sin t + 5$ and that $h(0) = 3$. Find the formula for $h(t)$. (15 points)

Read Carefully!! There are seven more problems, each worth fifteen points. Work on any **five** of them. If you work on more than five, you will get credit for the best five.

3. A tank of water is draining from a hole in the bottom of the tank. Torricelli's Law says that the depth of the water changes at a rate proportional to the square root of the depth. If $D(t)$ is the depth of the water at time t, write a differential equation that expresses this law.

4. Determine if $y(t) = t^2 - 3$ is a solution of the differential equation $t^3 y' = (y+3)^2$.

5. What can you say about the values of the following three limits? Justify all conclusions.

   (a) $\lim_{x\to 3^+} \dfrac{x-5}{x^2-x-6}$ (b) $\lim_{x\to 3^-} \dfrac{x-5}{x^2-x-6}$ (c) $\lim_{x\to 3} \dfrac{x-5}{x^2-x-6}$

6. Let $f(x) = 1/x$. Use the definition of derivative to find $f'(5)$.

7. F is an antiderivative of the function f shown. Assume that $F(4) = 3$ (OZ, second edition, pg 120, lower left).

   (a) What is the equation of the line tangent to the graph of F at the point where $x = 4$? (5 points)

   (b) Use the Speed Limit Law to give upper and lower estimates for $F(0)$. More specifically, give reasonably good (based on the graph) values for L and U such that you are sure that $L \le F(0) \le U$. Explain. (Make it clear how you are using the graph. Detailed algebra will suffice.) (10 points)

8. Find values for the constants A and B such that the function $y(x) = A\cos x + B\sin x$ satisfies $y(\pi/4) = 2$ and $y'(\pi/4) = 0$.

9. Find the maximum and minimum values of $f(x) = x^3 - 12x + 6$ on the interval $[0, 3]$.

Selected partial answers and hints.

2. $h(t) = e^t - 2\cos t + 5t + 4$

3. $D'(t) = k\sqrt{D(t)}$ or $dD/dt = k\sqrt{D}$. The question does not ask you to solve the differential equation.

4. Express the left and right sides of the equation *separately* as functions of t. Otherwise it's too easy to lose track of what you are trying to do. The two sides are not equal, and so the given function is not a solution.

5. Dividing top and bottom by x^2 is not helpful here. That only helps if $x \to \pm\infty$. The first limit is $-\infty$ and the second is ∞. The most you can say about the third is that it doesn't exist.

7.(b) $-29 \le F(0) \le -17$

8. $A = B = \sqrt{2}$

9. The maximum value is 6 and the minimum value is -10. Note: The question could have been "determine where the maximum and minimum values occur," in which case the answer would have been different. Knowing how to answer the question is worth some credit.