Math 111 **Exam 1** **Name:**

19 September 2005 100 Points

No calculators or *Mathematica*, except as indicated.

"Show enough work to justify your answers."

1. Consider the following statement: *"Gas prices have been going up, and now they are going up even faster."* Let $P(t)$ denote the average price of gas as a function of time over the time period suggested by the statement. Circle the conclusions that can be made from this statement. No justifications are necessary. Note: You should circle **all** the conclusions that can be made, and **only** those that can be made. (10 points)

P is	positive	negative	increasing	decreasing	concave up	concave down
P' is	positive	negative	increasing	decreasing	concave up	concave down
P'' is	positive	negative	increasing	decreasing	concave up	concave down

2. Let θ be the pictured angle.

   (a) What is the radian measure of θ? (4 points)

   (b) What are the exact values of $\sin\theta$ and $\cos\theta$? (6 points)

3. Solve for x: $2^{3x-2} = 5$ (10 points)

4. The graph of a cubic polynomial f is pictured. What is the formula of f? Two important things to note: 1) The directions the graph goes for large positive and negative values of x, and 2) The value of $f(0)$. You may leave your answer in factored form. (10 points)

5. Two copies of the graph of a function f are shown. Define functions g and h by $g(x) = -f(x)$ and $h(x) = f(x-2)$. Draw the graph of g on the left graph and the graph of h on the right graph. (10 points)

6. What is the natural domain of $f(x) = \dfrac{\sqrt{10-x}}{x}$? (10 points)

7. On the computer go to our course folder: Courses on Caleb N:/Math/Math111/Foote. Open the *Mathematica* file Exam1.nb and follow the instructions. Write your answers below. Important note: All I have to go on are the numbers you write, so partial credit is going to be nearly impossible. (10 points)

   Maximum and minimum x-values:

   Maximum and minimum y-values:

   Approximate value of $f'(6)$:

8. In the left column are graphs of five functions. In the right column are graphs of their derivatives, in a different order. Next to each letter, fill in the blank with the number of the derivative graph that goes with the graph for that letter. No justifications needed. (10 points)

   (a) _____

   (b) _____

   (c) _____

   (d) _____

   (e) _____

9. The following questions concern a function f whose derivative is shown below (OZ, second edition, page 62, bottom). **The graph of f is not shown.** (20 points)

   (a) Does f have a local minimum? If so, at what x-value? Explain. (5 points)

   (b) Where is f concave up? How do you know? (5 points)

   (c) Suppose $f(1) = 4$. Find an equation for the line tangent to the graph at $x = 1$. (5 points)

   (d) In the blank, write the smallest possible number that makes the statement true:

   $f'(x) \leq$ _____ for $-1 \leq x \leq 1$ (1 points)

   (e) Suppose $f(1) = 4$. Use your answer from the previous part and the Speed Limit Law to explain why $f(-1) \geq -4$. (4 points)

   This is the graph of f'. **The graph of f is not shown.**

Selected partial answers and hints.

1. The only thing you can say about P'' is that it is positive.

2. Take ln or $\log_2$ of both sides. Note: As part of an answer "ln" and "$\log_2$" don't make sense, since they are incomplete. They have to be applied to something, for example, "$\ln 5$" and "$\log_2 5$".

4. The initial guess, obtained from the roots, has the wrong value for $f(0)$ and goes to infinity in the wrong directions. It's tempting to add a constant, but that changes the roots. Modify it in some other way.

6. x can be any number less than or equal to 10 except 0.

9.(b) f is concave up when $-4 \le x \le -1$ and when $3 \le x \le 4$.

9.(c) The slope of the line is 2. The best form for the answer is point-slope form.

9.(d) $f'(x) \le 4$ for $-1 \le x \le 1$

9.(e) $f(-1) \ge -4$