**Math 111 (Foote)** **Exam 2** **Foote**

4 October 2000 100 Points

*"Show enough work to justify your answers."*

1. Let $f(x) = x^2 - 3x + 5$.
   a) Use the definition of the derivative learned in class to compute $f'(-2)$. (10 points)
   b) Give an equation of the line tangent to the graph at $x = -2$. (5 points)

2. The three graphs pictured below are of a function $f$ and its first two derivatives $f'$ and $f''$. Determine which graph goes with which function and briefly explain how you know. (15 points)

3. Suppose that $f$ is a function that is concave down on the interval $0 \le x \le 2$.
   a) Explain why $\big(f(1.1) - f(1)\big)/.1$ is an approximation for $f'(1)$. (8 points)
   b) Is the fraction in part a) an overestimate or underestimate of $f'(1)$? Why? (It may help to draw a picture.) (7 points)

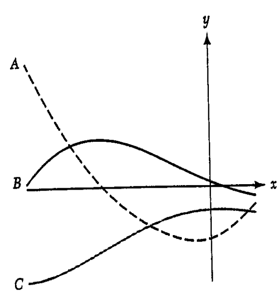
4. Suppose $f'(x) > -2$ for $1 \le x \le 8$ and that $f(5) = 3$. Use the Racetrack Principle to say something about the values of $f(1)$ and $f(8)$. To get full credit you must define and use an auxiliary function $g$ to compare to $f$. (15 points)

5. A submarine is stalking a battleship in a large, shallow bay. At the command of the captain, the submarine shoots a torpedo at the battleship. Starting at rest, the torpedo quickly gets up to its cruising speed of 25 feet per second. The aim of the torpedo, however, is not good, and it misses its target. After traveling at its cruising speed for three minutes, the torpedo's propulsion system runs out of fuel. The torpedo gradually loses speed due to water resistance and comes to rest on the floor of the bay five minutes after it was fired.

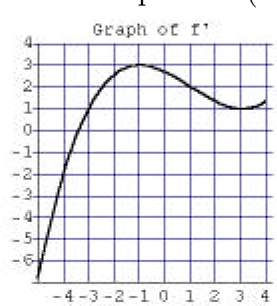
   Let $f(t)$ be the distance traveled by the torpedo as a function of time $t$ in seconds after it is fired. Sketch the graphs of $f$ and $f'$ for $0 \le t \le 300$. Clearly indicate the parts of your graphs related to the significant parts of the story, in particular, when the torpedo reaches its cruising speed, when it runs out of fuel, and when it stops. (10 points)

6. The graph of the derivative of a function $f$ is shown below. (15 points)
   a) On what interval(s) is $f$ decreasing? How do you know?
   b) Where (at what values of $x$) does $f$ have local minima? How do you know?
   c) Where does $f$ have inflection points? How do you know?

7. The graph of the derivative of a function $f$ is shown (same as previous problem). Explain why $f(4) - f(-3) > 11$. If you are unsure about how to do this, you may explain why $f(4) - f(-3) \ge 7$ for partial credit of 8 points. You may do both, in which case you will get credit for the explanation worth the most points. (15 points)




Problem 2




Problems 6 and 7

Selected answers and hints.

1. a) $f'(-2) = -7$ b) The line has to go through the same point as the function at $x = -2$.
2. Note that the function with graph $C$ is always negative.
3. Think about this in terms of slopes or the definition of derivative. Note that the fraction is *not* the definition of derivative.
4. One way is to let $g$ be any function with $g'(x) = -2$. The other way is to let $g$ be the function with $g'(x) = -2$ and $g(5) = 3$ (use the point-slope formula to get the formula for $g$ in this case). These use different versions of the Racetrack Principle, but give the same answers. $f(1) < 11$ is part of the answer. The other part is a lower bound on $f(8)$.
5. The torpedo is always moving away from the submarine. What does this say about the graphs of $f$ and $f'$?
6. $f$ does not always have a local max or min when $f'$ is zero. $f$ can have a local max or min at a point where $f'$ is not zero. $f$ does not always have an inflection point when $f''$ is zero. $f$ can have an inflection point at a point where $f''$ is not zero. It's good to have examples of these in mind when thinking about problems like this one.
7. Break the interval $[-3, 4]$ into pieces and use the RTP on each piece. Remember, it's not enough to say what $f'$ does at the endpoints of each piece.