Dr. Foote's Algebra & Calculus Tips
or
How to make life easier for yourself, make fewer errors, and get more points

This handout includes several tips that will help you streamline your computations. Most of the things it suggests avoiding are not technically wrong (that is, they won't cause you to lose points by themselves), but failing to avoid them will slow you down or prevent you from making needed simplifications (which *will* cause you to lose points). *You can improve some of your mathematical ability just by paying attention to a few simple things.*

Sometimes your own bad habits are your biggest obsticals to success.

Algebra Tips

- Be aware of the algebra errors you routinely make and watch out for them. Identify them on the Deadly Sins of Algebra handout.

- When you simplify things, be sure to minimize the agony and not maximize it.

  Don't: (Multiply things only to have to factor them later) $\frac{12}{5} \cdot \frac{2}{3} = \frac{24}{15} = \frac{8 \cdot 3}{5 \cdot 3} = \frac{8}{5}$

  Do: (Cancel factors *before* you multiply) $\frac{12}{5} \cdot \frac{2}{3} = \frac{4}{5} \cdot \frac{2}{1} = \frac{8}{5}$

  Don't: $9^{3/2} = \sqrt{9^3} = \ldots \text{(dig through backback for calculator)} \cdots = \sqrt{729}$
  $= \ldots \text{(use calculator again)} \cdots = 27$

  Do: (Take the square root *first*.) $9^{3/2} = (\sqrt{9})^3 = 3^3 = 27$

- Cancel things that easily cancel; combine things that easily combine.

$$\cdots + 2x + \cdots - y + \cdots - x + \cdots + 4y + \cdots - x + \ldots$$

should become

$$\cdots + 3y + \ldots$$

This is such an obvious thing to do, yet I often see uncombined or uncancelled quantities carried from one line to the next, often for several lines.

- Use exponents instead of roots.

$$\frac{x}{\sqrt[3]{x^2}} \text{ is hard to simplify.}$$

$$\text{Write it as } \frac{x}{x^{2/3}} \text{ instead. Then it simplifies easily to } x^{1/3}.$$

- Here is the best way to simplify compound fractions: multiply top and bottom by the least common multiple of the denominators. Multiply only as much as is needed to clear the fractions. Keep the rest factored.

$$\frac{\frac{1}{x} - \frac{1}{a}}{x-a} = \frac{\frac{1}{x} - \frac{1}{a}}{x-a}\,\frac{ax}{ax} = \frac{a-x}{(x-a)ax} \qquad\qquad \frac{\frac{3}{x} + 2}{4 - \frac{x}{2}} = \frac{\frac{3}{x} + 2}{4 - \frac{x}{2}}\,\frac{2x}{2x} = \frac{2(3+2x)}{x(8-x)}$$

- Get beyond some of the algebraic "helper steps" you were taught. To solve for y in the following, skip the steps where you "indicate" what you are doing to both sides of the equation.

$$\begin{aligned} 2y - 8 &= 3x \\ 2y - 8 + 8 &= 3x + 8 && \text{Skip this step!} \\ 2y &= 3x + 8 \\ \frac{2y}{2} &= \frac{3x+8}{2} && \text{Skip this step!} \\ y &= \frac{3x}{2} + 4 \end{aligned}$$

If you include all of the helper steps in a complicated problem, you will waste a lot of time!

- The opposite of the previous tip is: Don't try to do too much in your head!

- Don't write $\frac{1}{2}x$ as $\frac{1}{2x}$. It looks like x is in the denominator. I see this a lot. Not only does it sometimes confuse me (and you don't want to confuse the grader!), I have seen it confuse the person writing it—the x has actually wound up in the denominator a few steps later.

- Keep expressions factored unless there is a reason to multiply them out. The factored form has more information in it. If something should cancel, you won't see it if it is multiplied out.

- If you get a negative answer for something that should be positive (like an area or length), it indicates that you have made a mistake. *Don't simply take the absolute value and assume it is correct!* Go back and find your mistake.

- Be organized in your computations.
  - **Do:** Put equals signs between things that are equal.
  - **Don't** put equals signs between things that aren't equal!
  - Use " $\Longrightarrow$ " for implications, not equality.
  - Don't cross things out when they cancel, or at least do it sparingly. It's often hard for me to tell what you are thinking.
  - *Give me a chance* to understand the flow of your thought and the steps of your computation. I shouldn't have to guess what you are thinking—it should be clear.

- Don't cross out a problem on an exam. You might lose partial credit.

Trig and Log Tips

- Don't convert between degrees and radians outside the first quadrant. *Learn to think in radians for the standard angles outside the first quadrant* (multiples of $\pi/6 = 30°$, $\pi/4 = 45°$, $\pi/3 = 60°$, and $\pi/2 = 90°$). You should be able to determine the location of the angle $17\pi/6$ by drawing a picture faster than you can compute how many degrees it is using a calculuator.

- Strive to really understand the trig functions. *This will pay off.* You don't *really* understand them until you can find the exact values of sine and cosine of the standard angles using the unit circle the the two basic triangles without memorizing tables or quadrants. You should be able to figure out $\sin(17\pi/6)$ and $\cos(17\pi/6)$ with pencil and paper faster than you can with a calculator—and more accurately too! Your calculator will only give approximations.

- Don't rely on your calculator for the values of $\ln 1$, $\ln e$, and $\ln 0$. Everyone (and I mean *everyone!*) has trouble understanding logs and exponentials at first. *Push yourself to understand them!* The sooner you do this, the sooner you will really begin to understand applications that use them (such as pH, electrostatic potential, and inflation).

- Know and use standard identities.

$$\text{Replace} \begin{cases} \sin^2 x + \cos^2 x \\ \ln e \\ \ln 1 \end{cases} \text{with} \begin{cases} 1 \\ 1 \\ 0 \end{cases}.$$

I have seen someone carry $(\ln 1)(x^2+1)$ several steps through to the end of a computation without realizing that it drops out because $\ln 1 = 0$. Here is another case (which I have seen more than once) where a simple observation could have saved a lot of work.

$$\begin{aligned}
\int (\sin x + \cos x)^2\,dx &= \int (\sin^2 x + 2\sin x\cos x + \cos^2 x)\,dx \\
&= \int \sin^2 x\,dx + 2\int \sin x\cos x\,dx + \int \cos^2 x\,dx \\
&= \frac{1}{2}\int (1-\cos 2x)\,dx + 2\int u\,du + \frac{1}{2}\int (1+\cos 2x)\,dx \\
&= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + u^2 + \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C \\
&= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + \sin^2 x + \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C
\end{aligned}$$

One student left this as his final answer. Another realized it could be simplified to $x + \sin^2 x + C$. Compare this with the intended solution:

$$\begin{aligned}
\int (\sin x + \cos x)^2\,dx = \int (\sin^2 x + 2\sin x\cos x + \cos^2 x)\,dx &= \int (1 + 2\sin x\cos x)\,dx \\
= \int 1\,dx + 2\int \sin x\cos x\,dx &= x + \sin^2 x + C
\end{aligned}$$

Even if you don't know the calculus involved, you can appreciate the fact that replacing $\sin^2 x + \cos^2 x$ with 1 makes a huge difference in how hard the problem is!

Calculus Tips

- Don't use the Quotient Rule when the top or bottom is constant.

$$\left(\frac{x^2}{3}\right)' = \left(\frac{1}{3}x^2\right)' = \frac{1}{3}\cdot 2x = \frac{2}{3}x \qquad\qquad \left(\frac{3}{x^2}\right)' = \left(3x^{-2}\right)' = -6x^{-3} = -\frac{6}{x^3}$$

- Instead of using the Quotient Rule, some people like to write $\frac{f(x)}{g(x)}$ as $f(x)g(x)^{-1}$ and then use the Product, Power, and Chain Rules. This is rarely easier, and it often leads to more complicated algebra.

- *Don't let algebra errors rob you of points on calculus problems.* Be aware of algebra errors you frequently make (everyone does) and go over your work looking for them. If you frequently lose points for algebra errors, ask your instructor to help you understand and avoid them.

- Once you get past the Basic Skills Exam, *don't let basic skills errors rob you of points on problems that use calculus.* For example, don't screw up a problem by misapplying the product rule or forgetting the chain rule.