STRATEGY FOR TAKING ANTIDERIVATIVES

We have studied several methods for evaluating antiderivatives: simple substitutions, trigonometric substitutions, using trigonometric identities or the method of partial fractions to modify the integrand, and integration by parts. After working the problems in these sections, you should be familiar with the steps involved with each of the methods.

With all of these methods we have a new problem: given an antiderivative to evaluate, how do you go about choosing which method to use? We need a selection procedure that

(1) is quick and takes little or no computation, and
(2) chooses the right method for the large majority of problems.

The key to choosing the right method is to observe the form of the integrand. (The integrand is the function that is to be integrated.)

⋆ You must really know (more than memorized!) the “Basic Antiderivative Formulas You Really Need to Know !!”

♢ Look for a function nested somewhere in the integrand so that its derivative (or a **constant** multiple of its derivative) is a factor of the integrand. This situation calls for a simple substitution. For example, in

$$\int \frac{2x+3}{x^2+3x-5}\,dx \qquad \text{we let} \qquad u = x^2+3x-5.$$

♣ If you have a combination of trigonometric functions, think about which trigonometric identities might help. Good identities to know:

$$\sin^2 x = (1-\cos 2x)/2, \qquad \cos^2 x = (1+\cos 2x)/2,$$
$$\sin^2 x + \cos^2 x = 1, \qquad \tan^2 x + 1 = \sec^2 x, \qquad 1+\cot^2 x = \csc^2 x,$$
$$\sin 2x = 2\sin x \cos x, \qquad \cos 2x = \cos^2 x - \sin^2 x.$$

♠ If the integrand has x^2+a^2, x^2-a^2, or a^2-x^2 in it (or if you can get this by completing a square), you may need a trigonometric substitution.

♯ If the integrand is a fraction and the numerator and denominator are both polynomials, you may need to use partial fractions. Be sure the degree of the top is less than the degree of the bottom (do long division if not), and factor the bottom. Take note of repeated and quadratic factors.

♡ If the integrand is the product of two functions that are not related at all, integration by parts may work. Let dv be the most complicated factor you can easily antidifferentiate.

Keeping these points in mind will allow you to choose the right method in the large majority of problems.

Some Do's and Don't's.

DO check every integral for an easy simple substitution. This doesn't mean try everything you can think of; it means see if there is an obvious one.

DON'T pass variables through the integral sign. This is especially tempting in a simple substitution. If you let $u = x^2$ and rewrite

$$\int \sin x^2\,dx \qquad \text{as} \qquad \frac{1}{2x}\int 2x\sin x^2\,dx,$$

you have made this mistake. When you choose u in a simple substitution, du can only be off by a constant factor or there must be a way to deal with the other terms containing x.

DON'T have any intermediate steps in a substitution with both variables present—it's too easy to get confused. Change completely from the old variable to the new.

DON'T go through the above list using brute force with each method trying to make it work. You can waste a lot of time if the method you need is at the end of the list. Let the form of the integrand lead you to the right method.

DO consider alternate methods. Some problems can be done in more than one way. Before jumping into a trigonometric substitution or partial fractions, see if a simple substitution will work. For example, in $\int \frac{x}{x^2+1}\,dx$, a trigonometric substitution will work, but it will take much longer to do than a simple substitution.

DO remember that some problems require more than one method. For example a simple substitution may change a problem into one requiring a trigonometric substitution. If you can see this coming, you may be able to combine the two substitutions into one.

DO consider breaking the integral up into a sum or difference of integrals if this is possible. This may be especially helpful if the integrand is a fraction, although it is not always appropriate. For example, breaking up the integral on the first page would make it worse, not better.

DO have a **reason** for choosing the method you use. With practice you can anticipate the results of an action before taking it. Before plowing into a long computation, try to determine if it is really going to help.

A good way to practice this strategy is to take a page of miscellaneous integrals (see the problems at the end of the chapter) and go through them deciding which method to use. Then go back and work them out, seeing if your choice was correct. Practicing this strategy should enable you to choose the right method for most problems within 15 seconds with little or no computation.