A COMMENT ON PARTIAL FRACTIONS

MATH 112

Partial fraction decomposition isn't a particularly deep subject, but there is one part of it that can be puzzling. Let's look at an example. Suppose we want to find the decomposition of $\dfrac{x}{(x+1)(x-1)}$. We should have

$$\frac{x}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \tag{1}$$

for some constants A and B. Multiplying by $(x+1)(x-1)$ we have

$$x = A(x-1) + B(x+1). \tag{2}$$

Letting $x = 1$ we get $1 = 2B$, so $B = 1/2$. Letting $x = -1$ we get $-1 = -2A$, so $A = 1/2$.

But wait! Is it really legitimate to let x be 1 or -1? After all, the left and right sides of (1) are equal for all values of x *except* 1 and -1! Therefore we know that the left and right sides of (2) are equal for all values of x *except* the ones we want to plug in!

Here's how to clean up this detail. The left and right sides of (2) are continuous functions. If two continuous functions have the same domain and are equal for all except perhaps a finite number of values of x, then they are equal for *all* values of x in their domain. Thus it really is okay to plug in any values we want.