Numerical Integration Error Estimates

Math 112, Section 7.7 of Stewart

Fall 2009

I like to write the error estimates in a way that conveys slightly more information than is done in our text.

- $I = \int_a^b f(x)\,dx$ This is the true value of the integral, often not accessible.
- $L_n,\ R_n,\ T_n,\ M_n,\ S_n$ The left endpoint, right endpoint, trapezoid, midpoint, and Simpson's approximations of I based on n subintervals.
- $I - R_n$ is the error made by the approximation $R_n \approx I$.
- $|I - R_n|$ is the magnitude of the error.
- K_i is any upper bound for the size of the i^{th} derivative of f on $[a, b]$, for example, $K_2 \geq |f''(x)|$ for $a \leq x \leq b$.

In the following, the right-hand sides are upper bounds for the errors, not the actual errors. In particular, even though the errors for the left and right endpoint approximations have the same upper bound, the errors themselves are generally not equal. In general, the error bounds represent the worst possible errors.

$$|I - L_n| \leq \frac{K_1(b-a)^2}{2n}$$

$$|I - R_n| \leq \frac{K_1(b-a)^2}{2n}$$

$$|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2}$$

$$|I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}$$

$$|I - S_n| \leq \frac{K_4(b-a)^5}{180n^4}$$

These inequalities are theorems. If you are interested, a proof for the trapezoid approximation is on the handouts page.

You do not need to memorize these. You *do* need to be able to use them and understand their meaning and consequences.